Finite Automata

Part Two

Recap from Last Time

Formal Language Theory

- An ***alphabet*** is a set, usually denoted Σ, consisting of elements called ***characters***.
- A ***string over Σ*** is a finite sequence of zero or more characters taken from Σ.
- The ***empty string*** has no characters and is denoted ε.
- A ***language over Σ*** is a set of strings over Σ.
- The language **Σ*** is the set of all strings over Σ.

DFAs

- A ***DFA*** is a
  - ***D***eterministic
  - ***F***inite
  - ***A***utomaton
- DFAs are the simplest type of automaton that we will see in this course.

DFAs

- A DFA consists of:
  - A set of states
  - Exactly one element of the set of states designated as a start state
    - (as a consequence, the set of states must be nonempty)
  - A subset of the states designated as accepting states
  - An alphabet Σ
  - A transition function that maps (state, character) ordered pairs to states
    - (i.e., for each state in the DFA, there must be ***exactly one*** transition defined for each symbol in Σ)

The Language of an Automaton

- If D is a DFA that processes strings over Σ, the ***language of D***, denoted $\mathscr{L}(D)$, is the set of all strings D accepts.
- Formally:

$$\mathscr{L}(D) = \{ w \in \Sigma^* \mid D \text{ accepts } w \}$$

New Stuff!

Recognizing Languages with DFAs

$L = \{ w \in \{\mathbf{a}, \mathbf{b}\}^* \mid w$ contains **aa** as a substring $\}$

Ask yourself these design questions:

What trait(s) of the string so far do I need to keep track of while processing?

For each trait, how many meaningfully distinct configurations of that trait are there that I need to keep track of?

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1 trait: count of consecutive a's seen so far

3 configurations: 0, 1, 2+

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Conclusion: we'll make 3 states: one each for 0, 1, 2+

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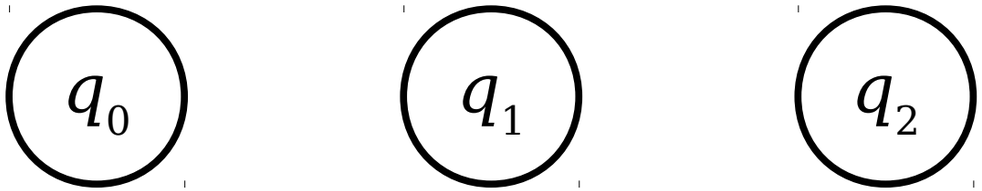
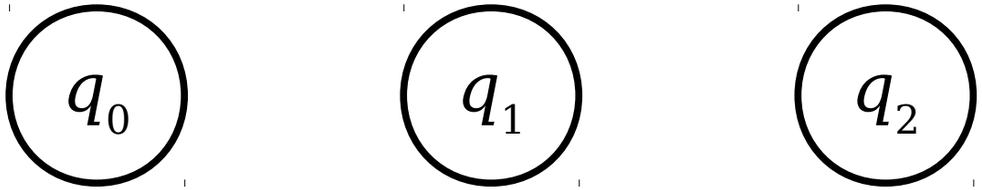


Conclusion: we'll make 3 states: one each for 0, 1, 2+

Recognizing Languages with DFAs

$L = \{ w \in \{\mathbf{a}, \mathbf{b}\}^* \mid w$ contains $\mathbf{aa}$ as a substring $\}$



Only after putting down all the states you'll need, think about what moves you from state to state, and connect them.

Recognizing Languages with DFAs

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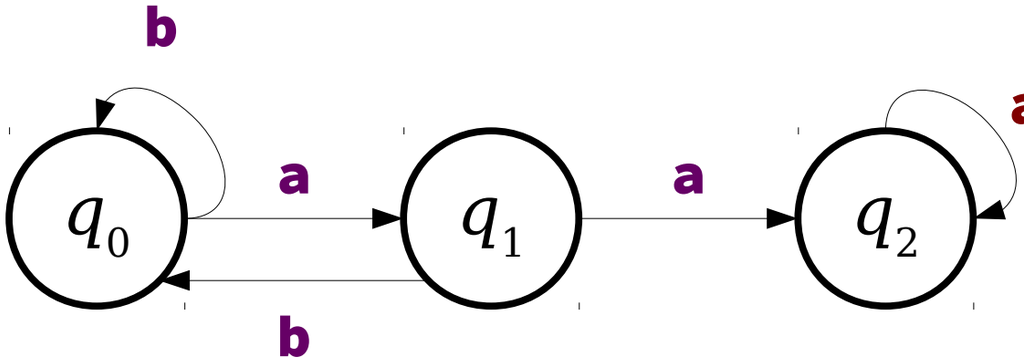

Seeing a letter **b** at the beginning changes nothing. Seeing an **a** means we advance our count. Seeing a **b** restarts our count. Once we've seen it, nothing changes our status.

Recognizing Languages with DFAs

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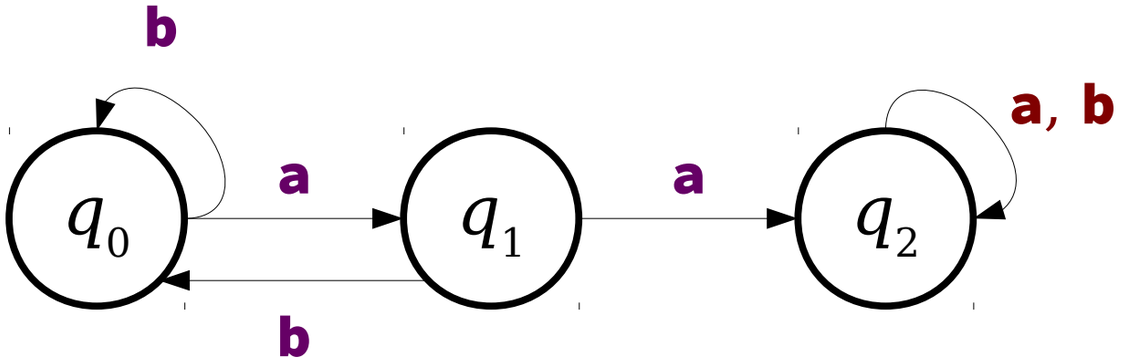

Now ask yourself: what is your status before you read any input? That configuration is our start state.

Recognizing Languages with DFAs

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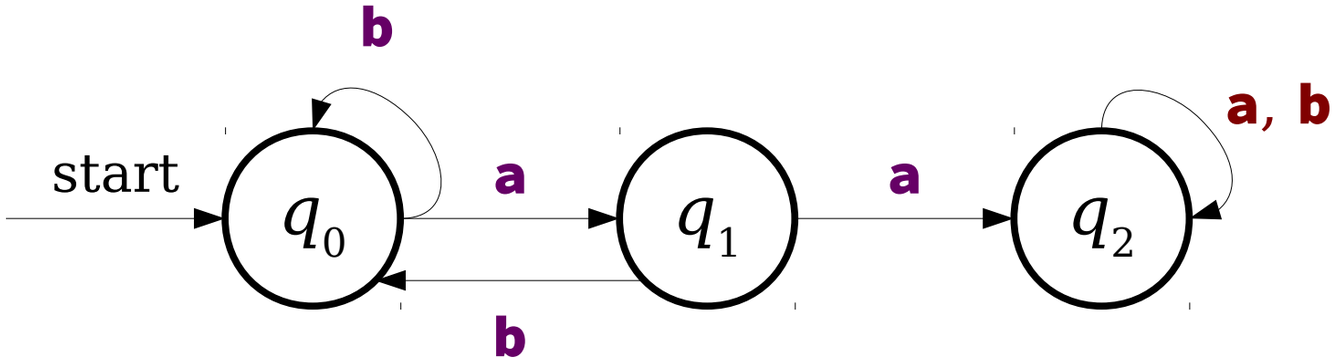

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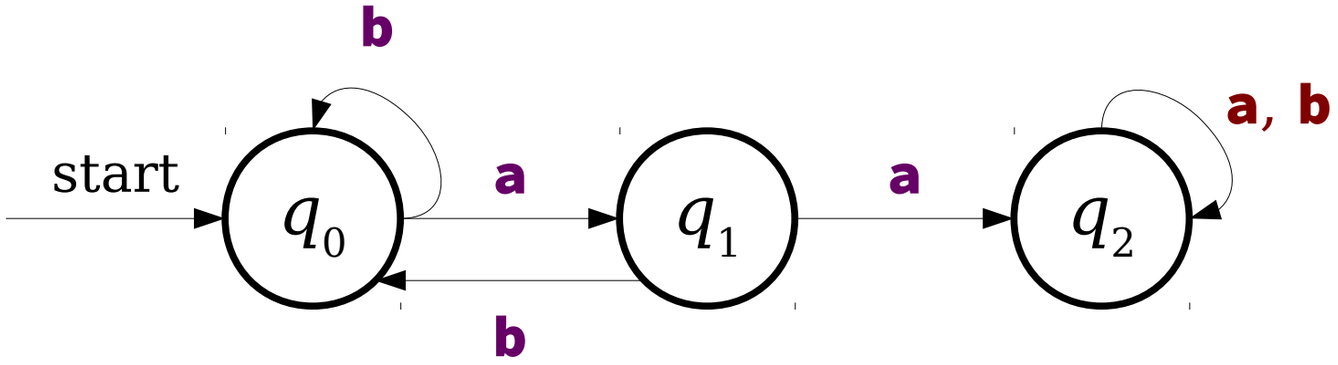

Now ask yourself: what status do we want to have at the end? Those configuration(s) are our accepting states.

Recognizing Languages with DFAs

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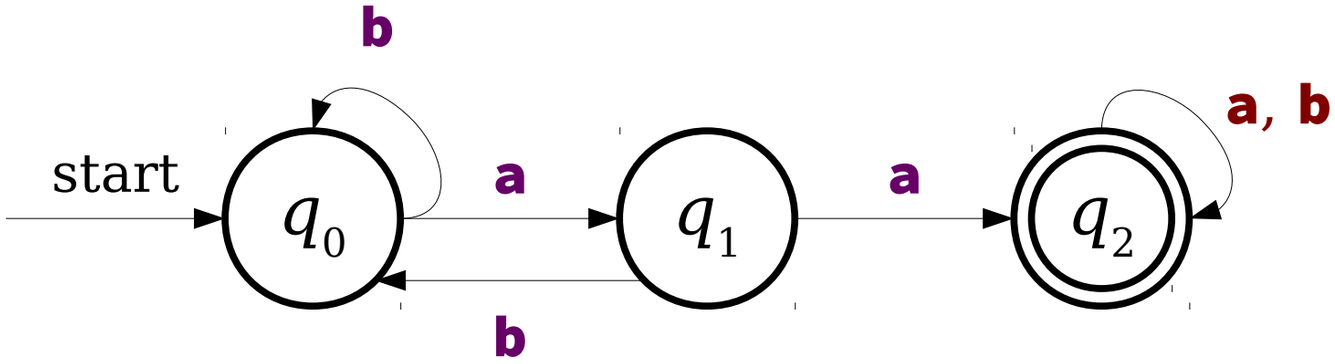

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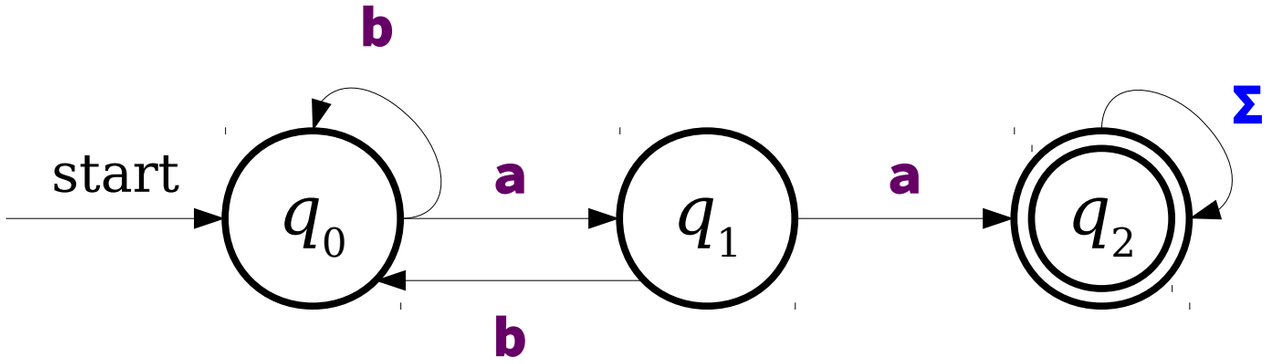

Style note: if a transition label includes every character in the alphabet, we can just use this shorthand instead of a long list of characters.

Recognizing Languages with DFAs

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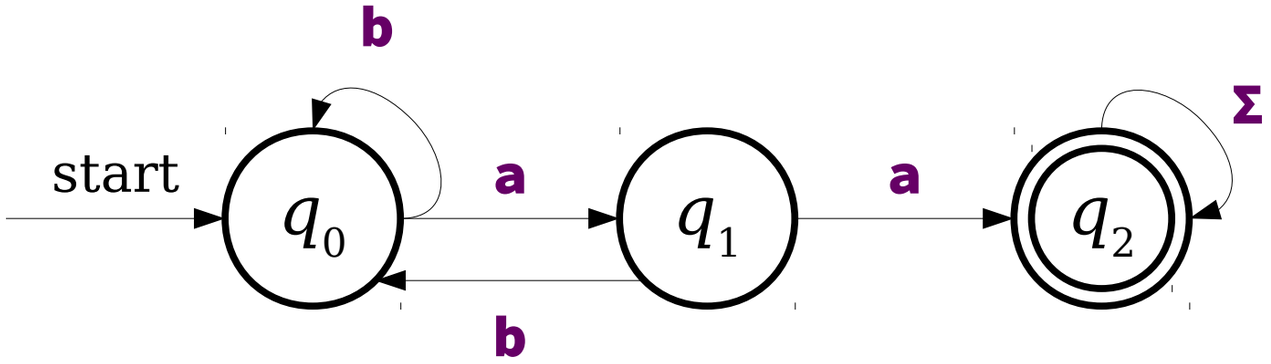

More Elaborate DFAs

$L = \{ w \in \{\mathbf{a}, \mathbf{*}, \mathbf{/}\}^* \mid w \text{ represents a C-style comment } \}$

Let's have the **a** symbol be a placeholder for "some character that isn't a star or slash."

Let's design a DFA for C-style comments. Those are the ones that start with /* and end with */.

Accepted:

/*a*/

//**

/*/**

/*aaa*aaa*/

/*a/a*/

Rejected:

/

//a/*aa*/**

aaa//aa**

/*/

/a/**

//aaaa

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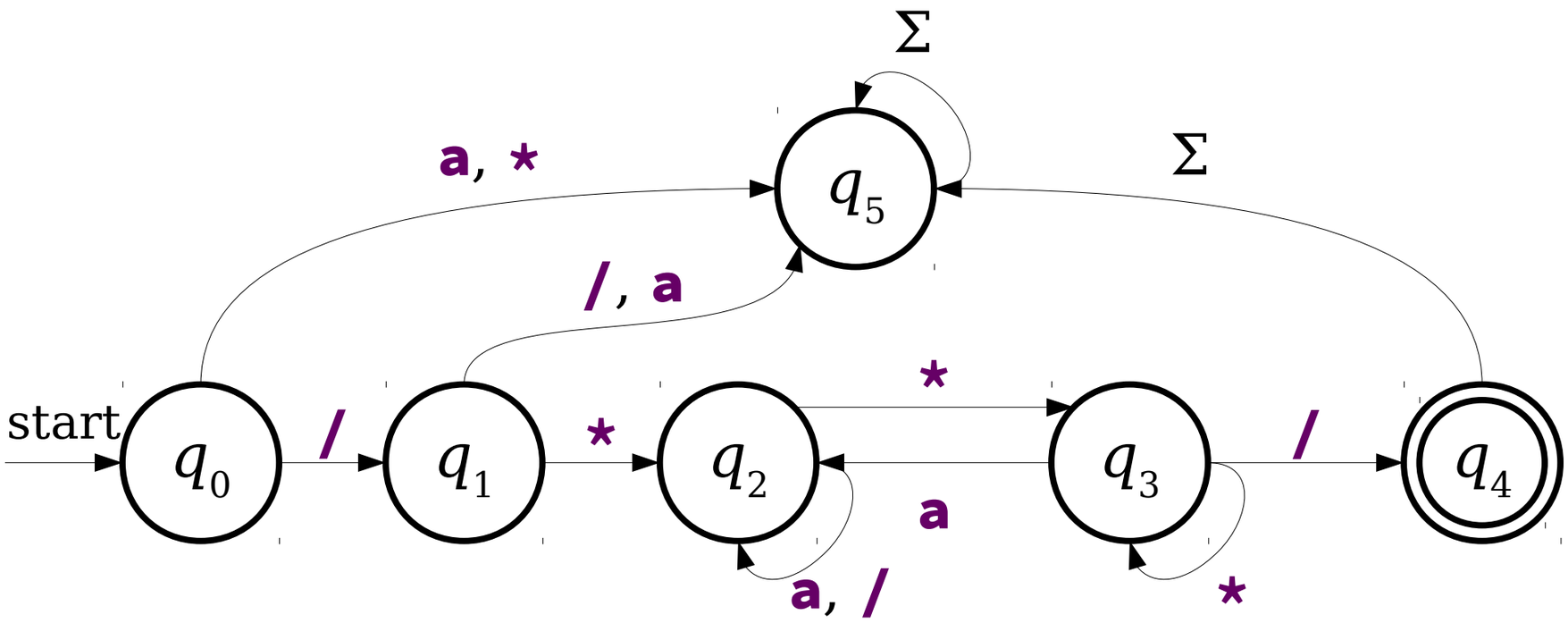
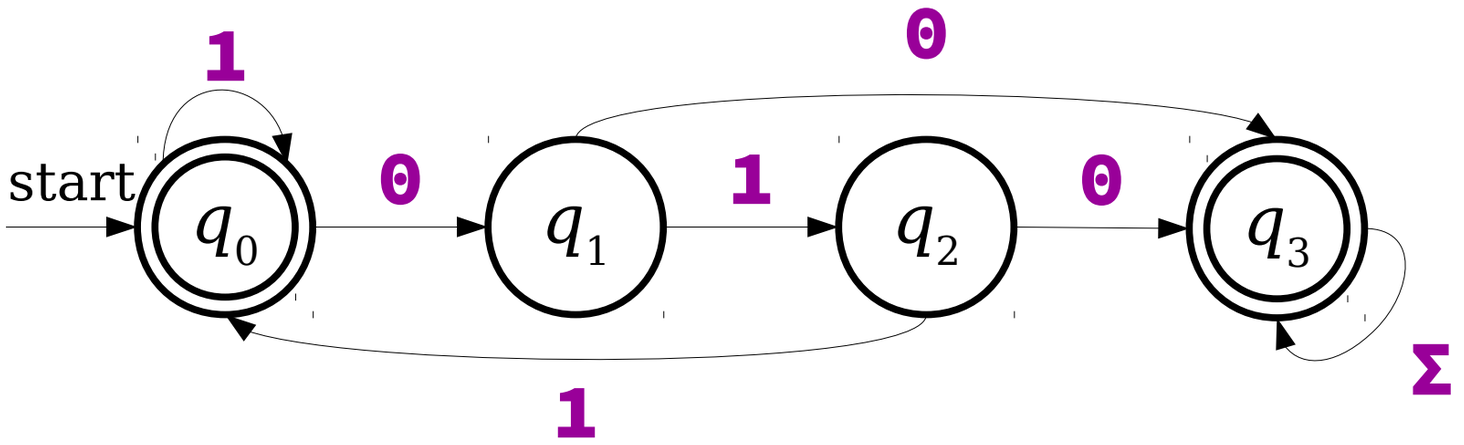


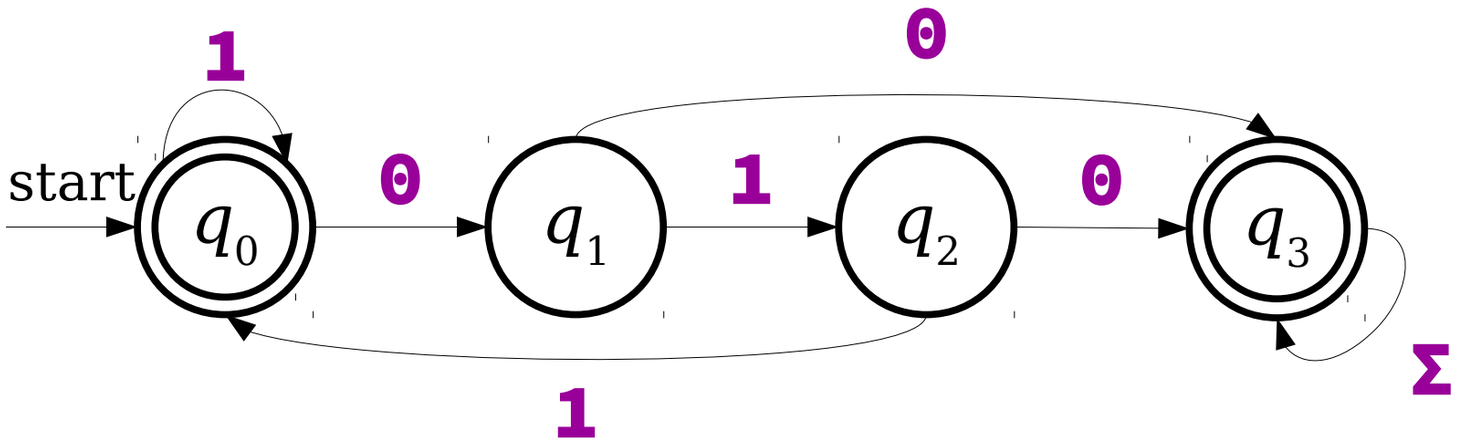
Table Representation of DFAs

Tabular DFAs



	0	1
q_0		
q_1		
q_2		
q_3		

Tabular DFAs



	0	1
q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
q_3	q_3	q_3

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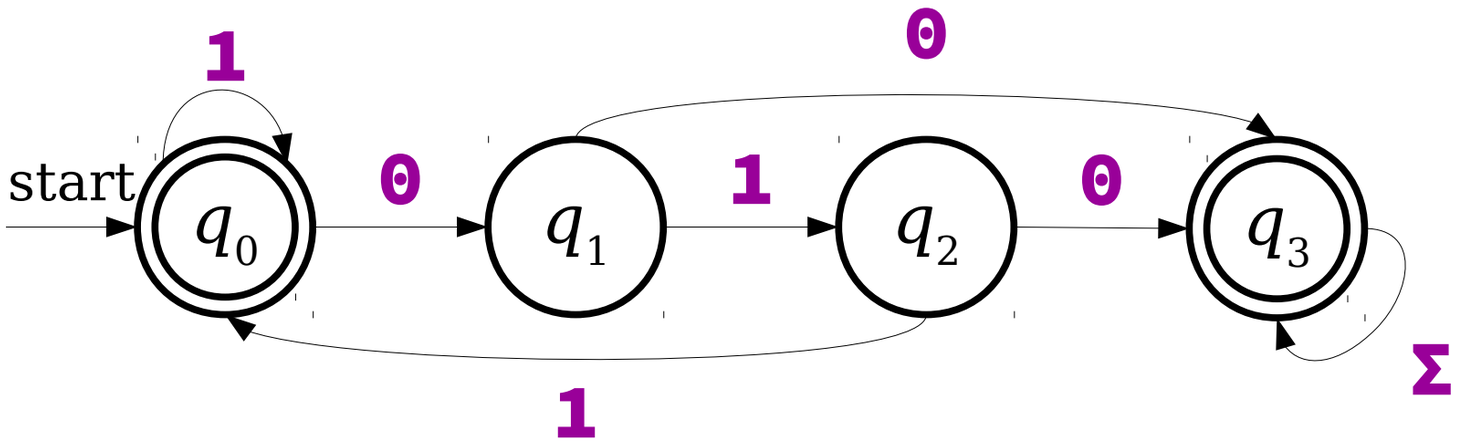
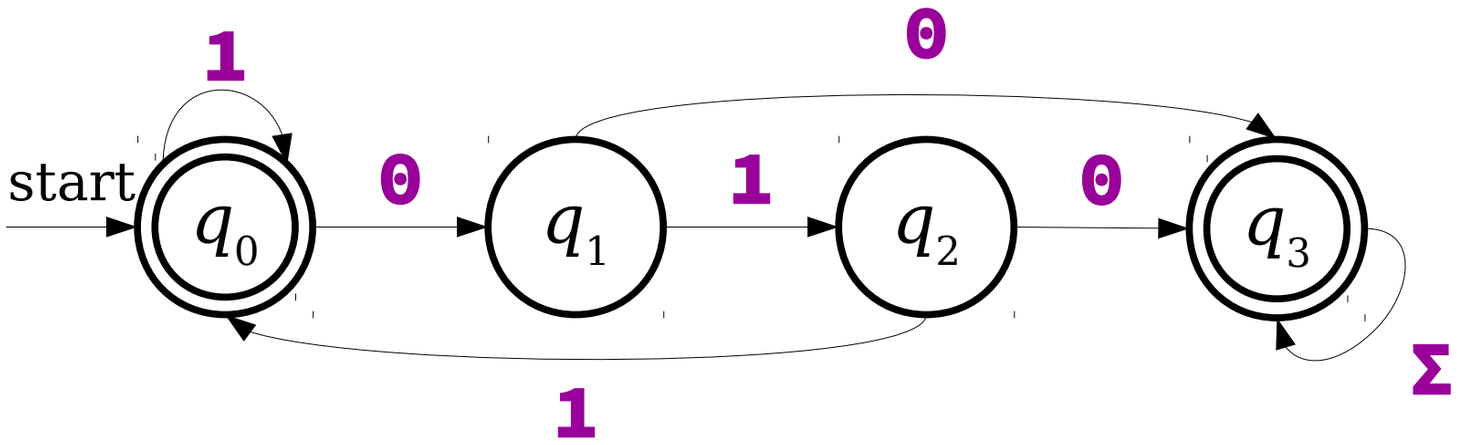


	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

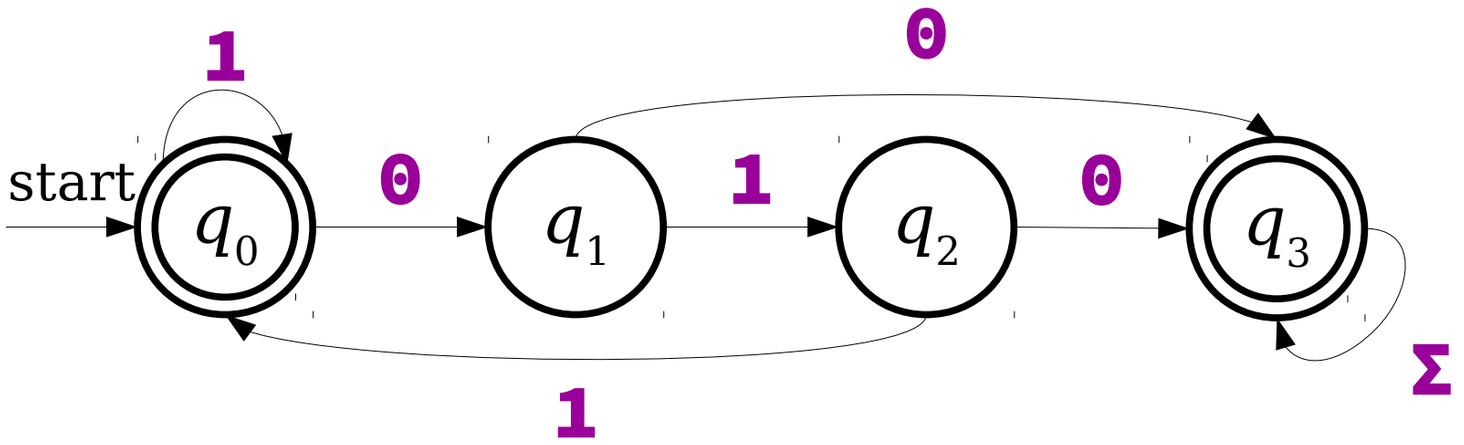
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

These stars indicate accepting states.

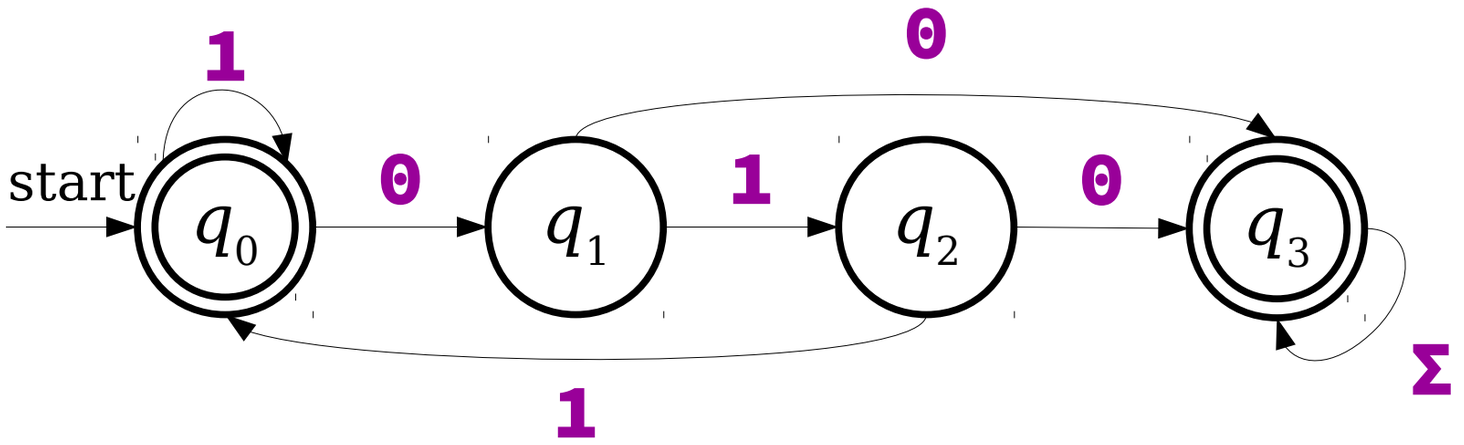
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Since this is the first row, it's the start state.

Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Question to ponder:
Why isn't there a column here for Σ?

The Regular Languages

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

If L is a language and $\mathscr{L}(D) = L$, we say that D ***recognizes*** the language L.

The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the ***complement*** of that language (denoted $\overline{L}$) is the language of all strings in Σ^* that aren't in L.
- Formally:

$$\overline{L} = \Sigma^* - L$$

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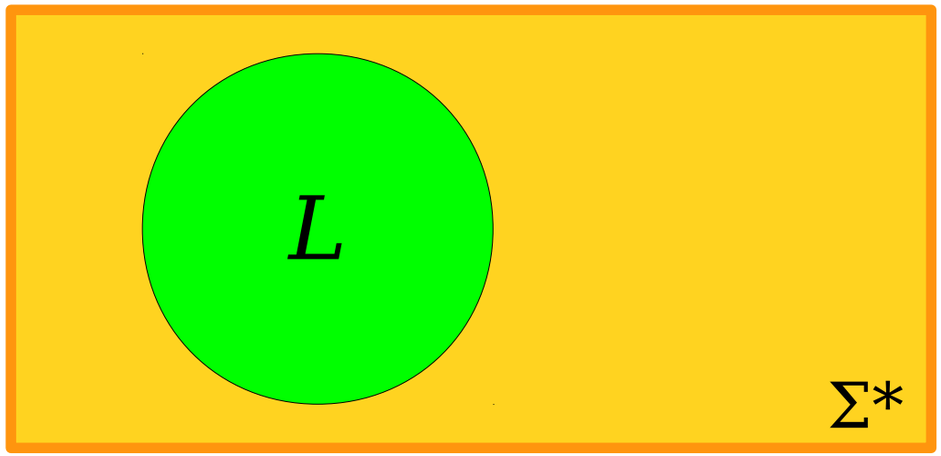

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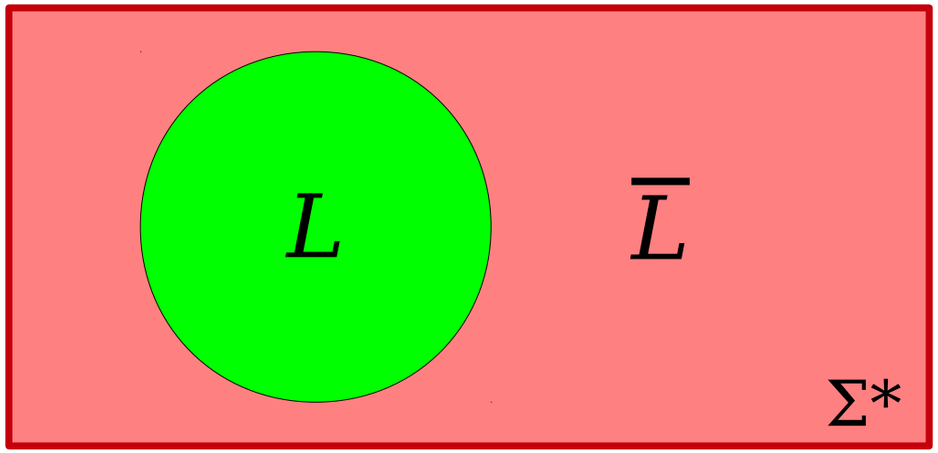

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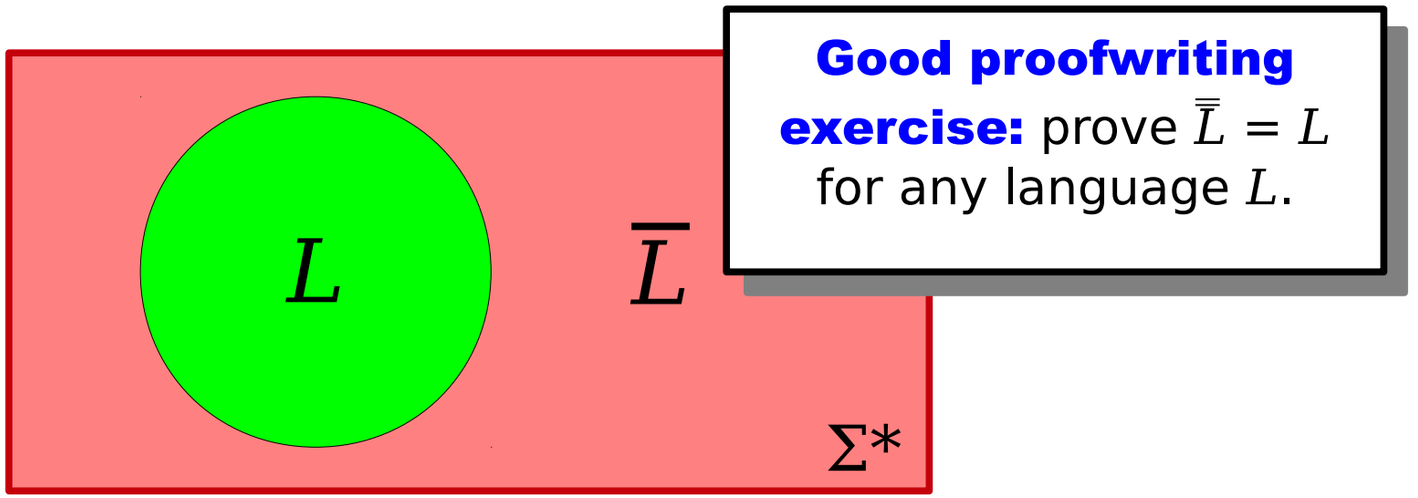
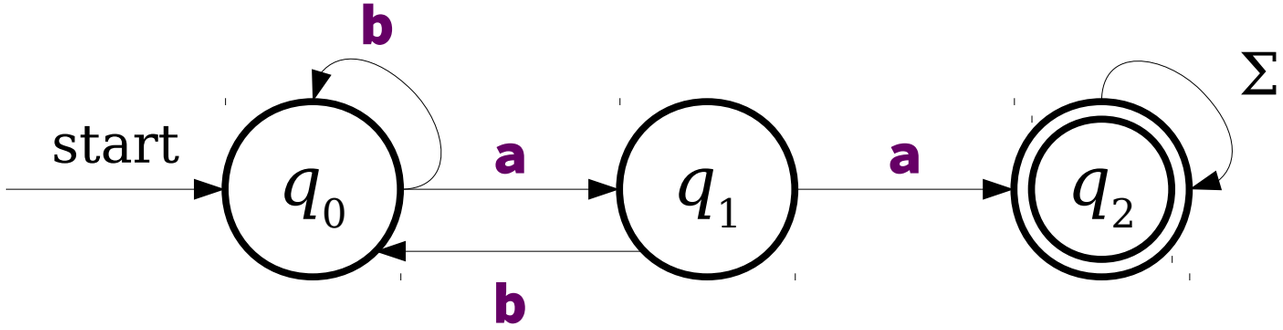


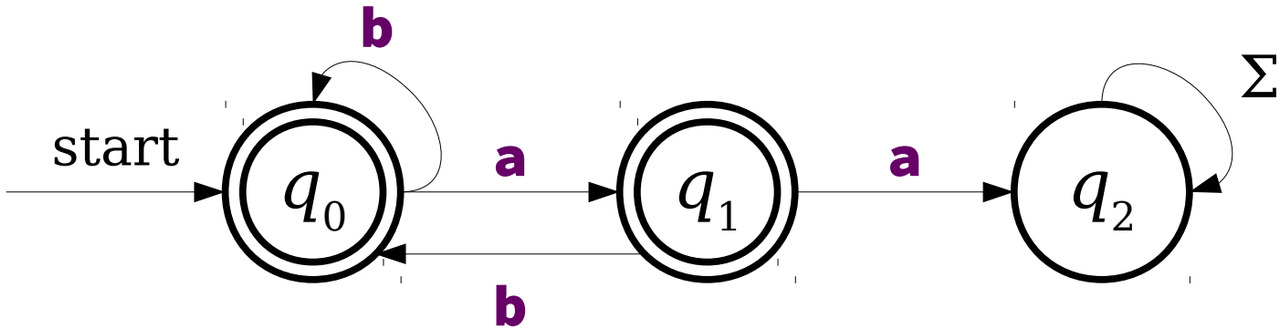
Good proofwriting exercise: prove $\overline{\overline{L}} = L$ for any language L.

Complementing Regular Languages

$L = \{ w \in \{\mathbf{a}, \mathbf{b}\}^* \mid w$ contains **aa** as a substring $\}$

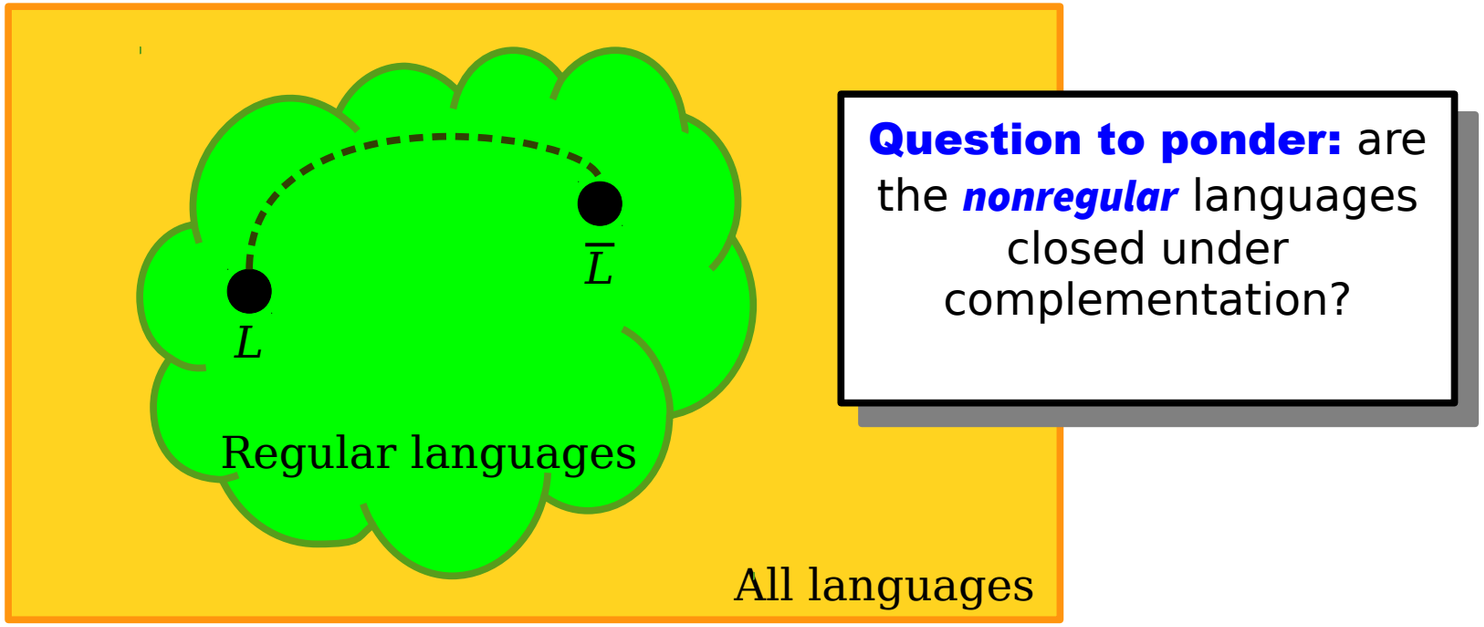


$\overline{L} = \{ w \in \{\mathbf{a}, \mathbf{b}\}^* \mid w$ ***does not*** contain **aa** as a substring $\}$



Closure Properties

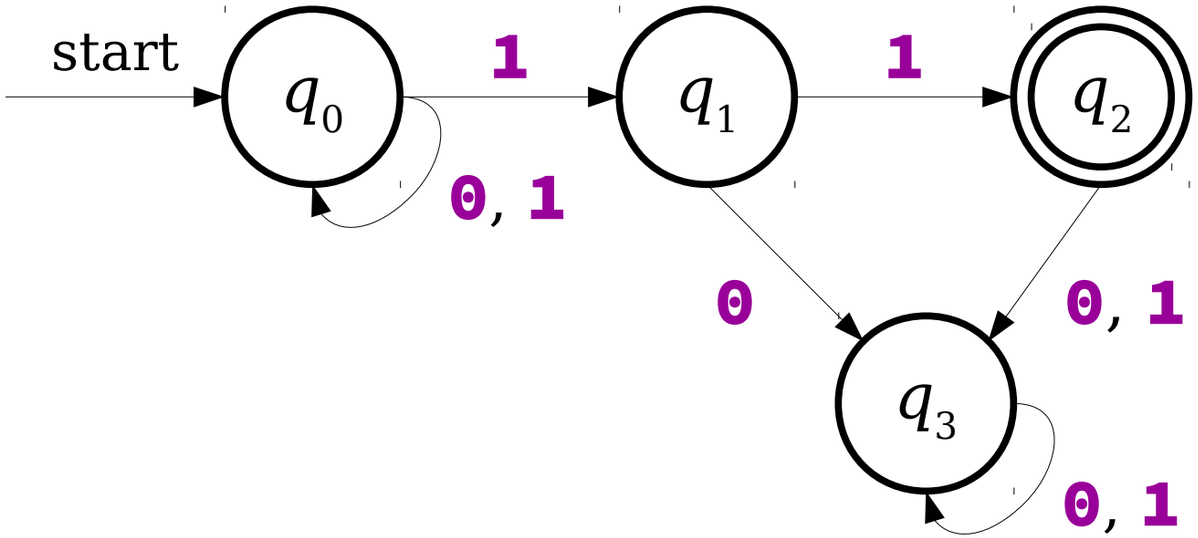
- ***Theorem:*** If L is a regular language, then $\overline{L}$ is also a regular language.
- (We haven't formally proved this, but you may assume it's true in this class.)
- As a result, we say that the regular languages are ***closed under complementation***.



Question to ponder: are the ***nonregular*** languages closed under complementation?

NFAs

Revisiting a Problem



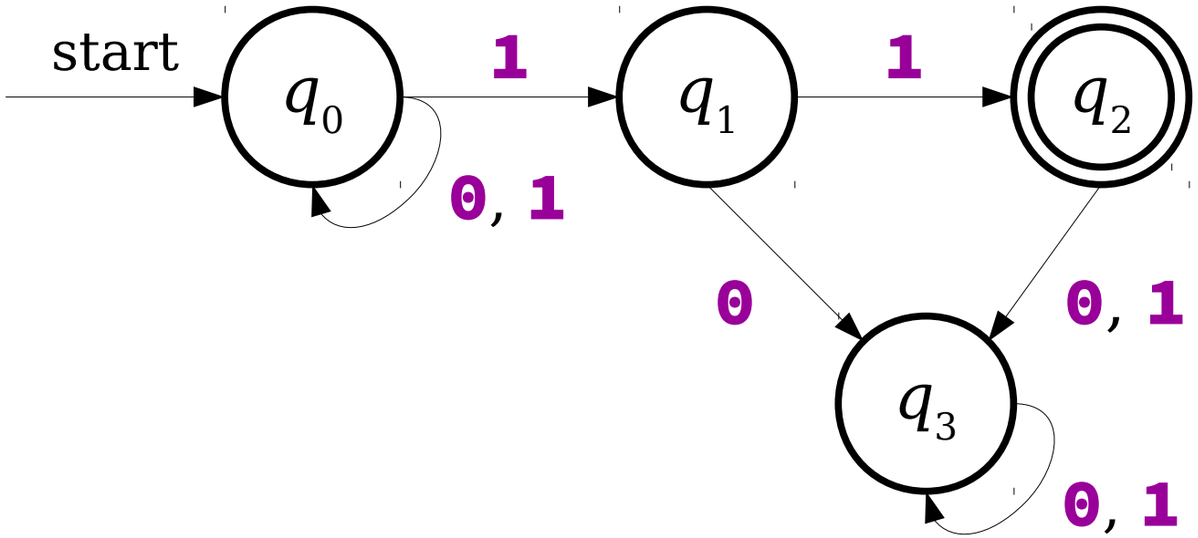
NFAs

- An ***NFA*** is a
  - ***N***ondeterministic
  - ***F***inite
  - ***A***utomaton
- Structurally similar to a DFA, but with a different transition function:
  - DFA: $\delta : (S \times \Sigma) \to S$ *always go to exactly one state*
  - NFA: $\delta : (S \times \Sigma) \to \wp(S)$ *could go to many, or none!*
- Represents a fundamental shift in how we'll think about computation.

(Non)determinism

- A model of computation is ***deterministic*** if at every point in the computation, there is exactly one choice that can make.
  - The machine accepts if that series of choices leads to an accepting state.
- A model of computation is ***nondeterministic*** if the computing machine has a finite number of choices available to make at each point, possibly including zero.
- The machine accepts if ***any*** series of choices leads to an accepting state.
  - (This sort of nondeterminism is technically called ***existential nondeterminism***, the most philosophical-sounding term we'll introduce all quarter.)

A Simple NFA



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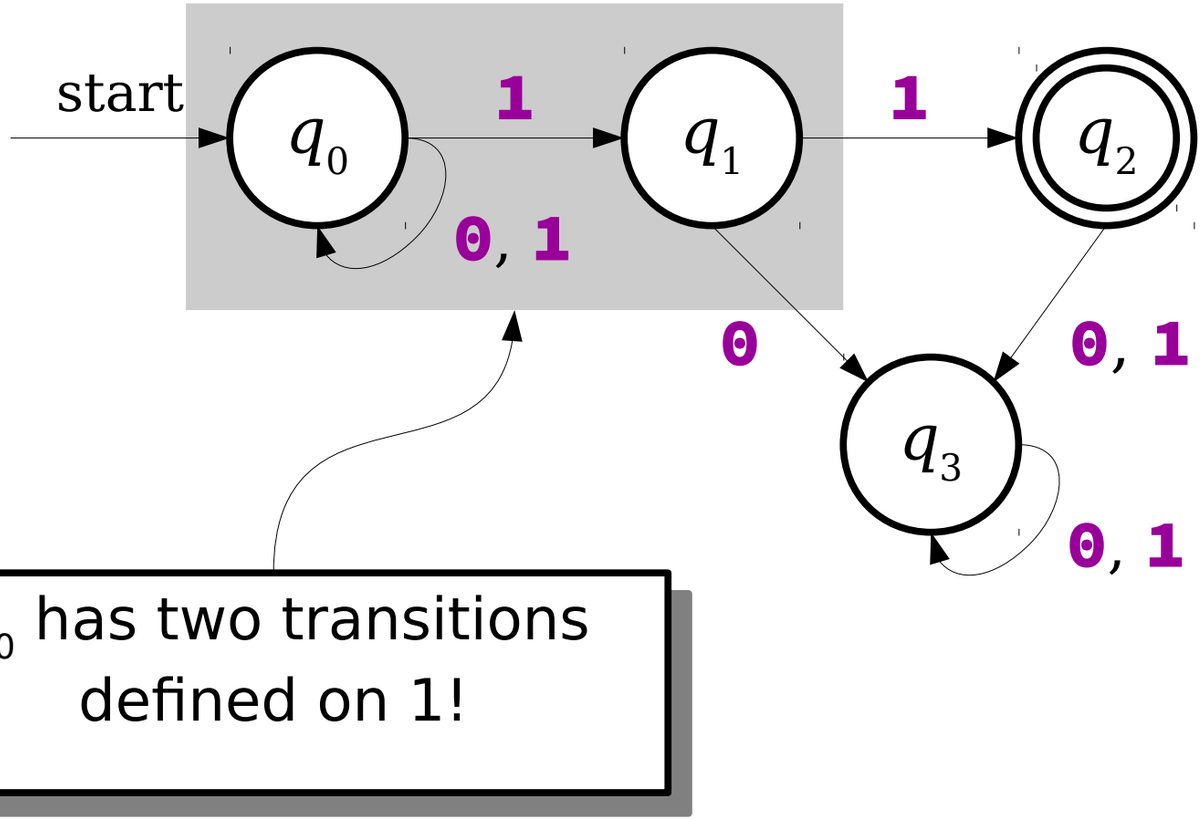
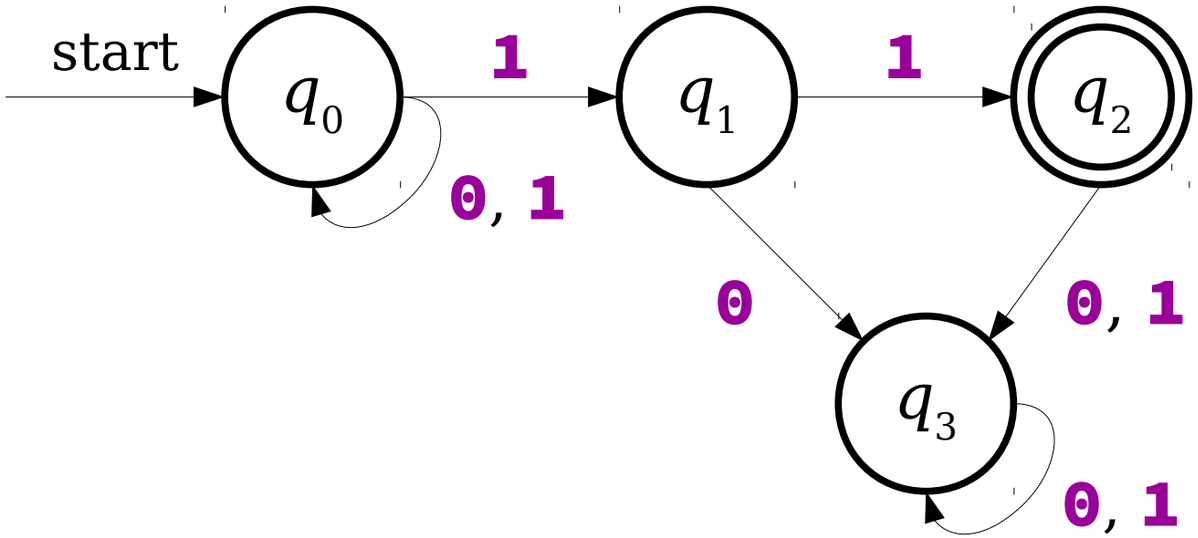


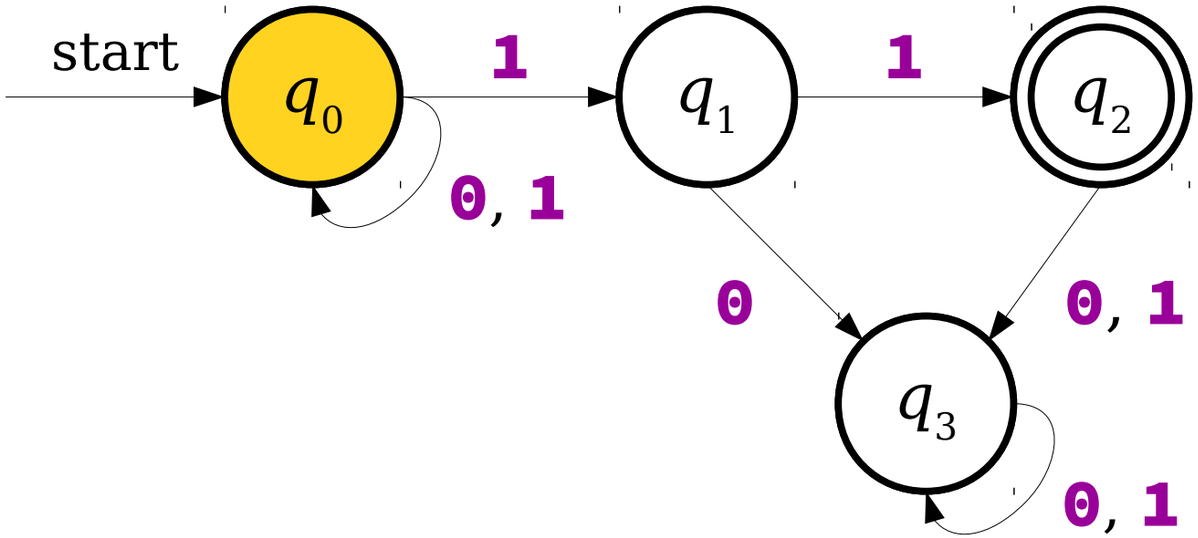
q_0 has two transitions defined on 1!

A Simple NFA



0	1	0	1	1

A Simple NFA



0	1	0	1	1

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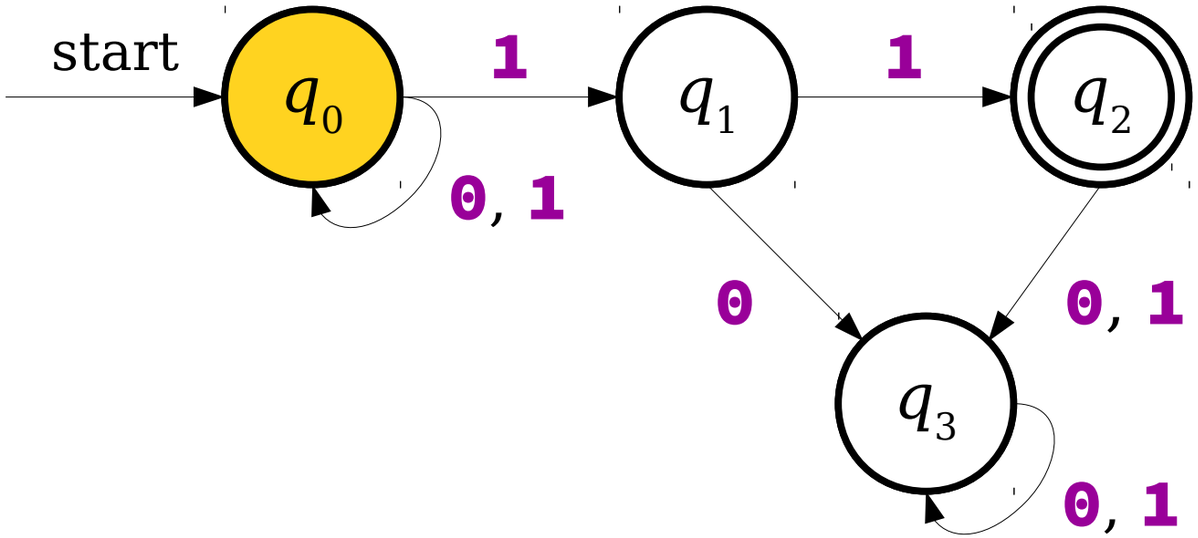

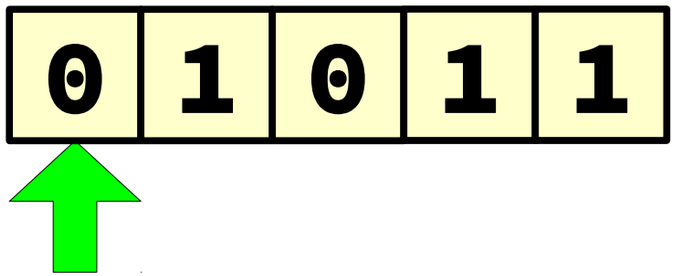

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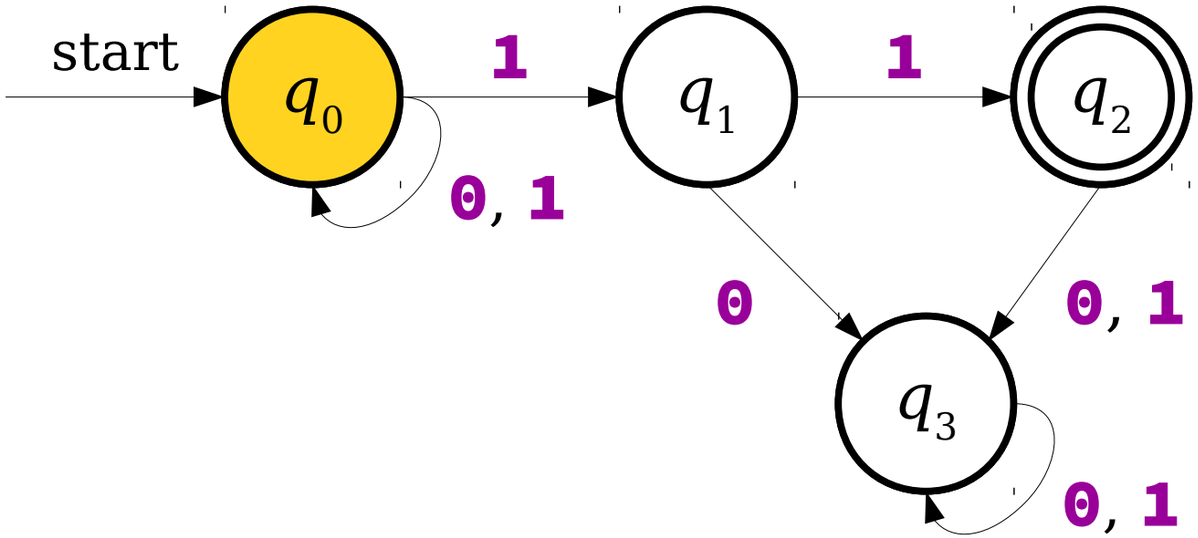

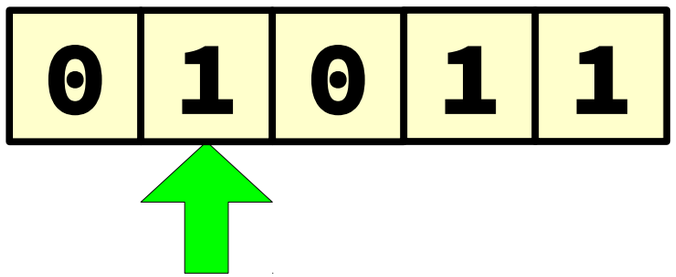

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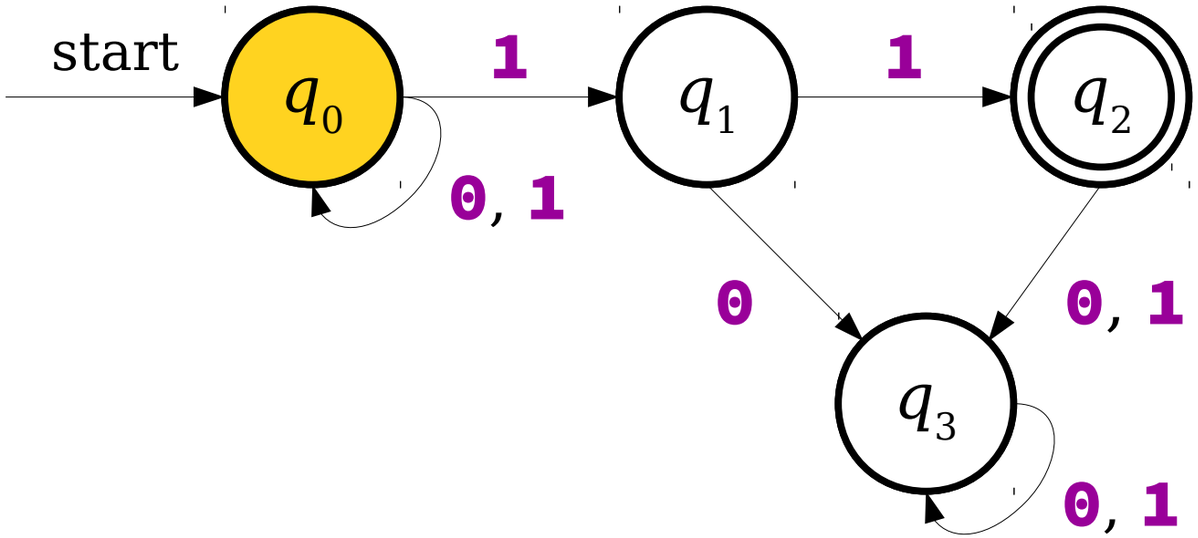

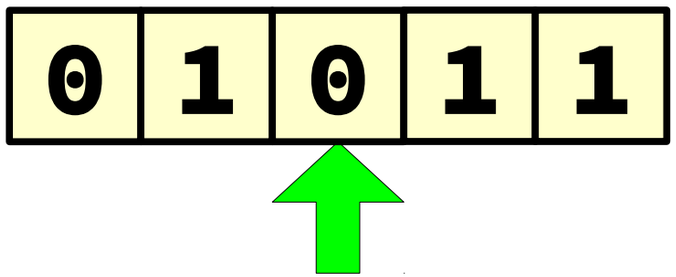

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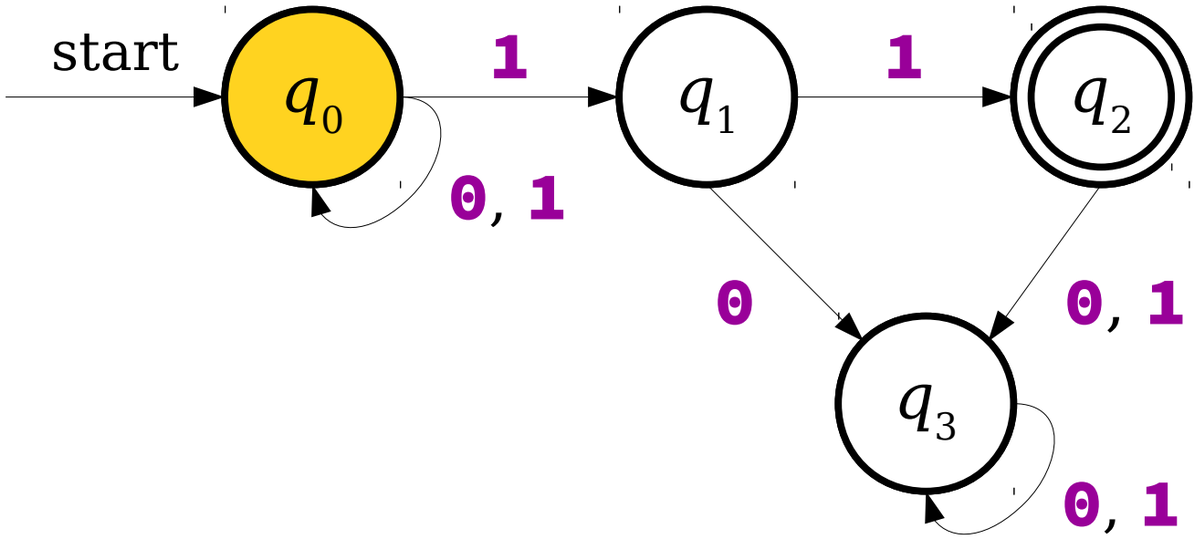

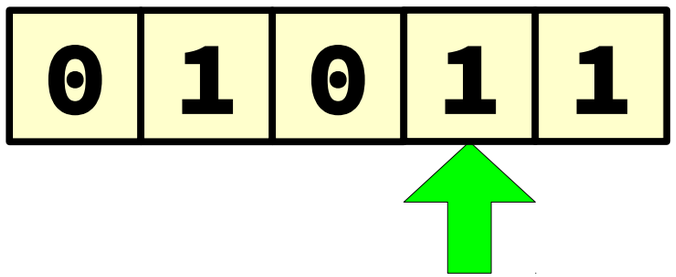

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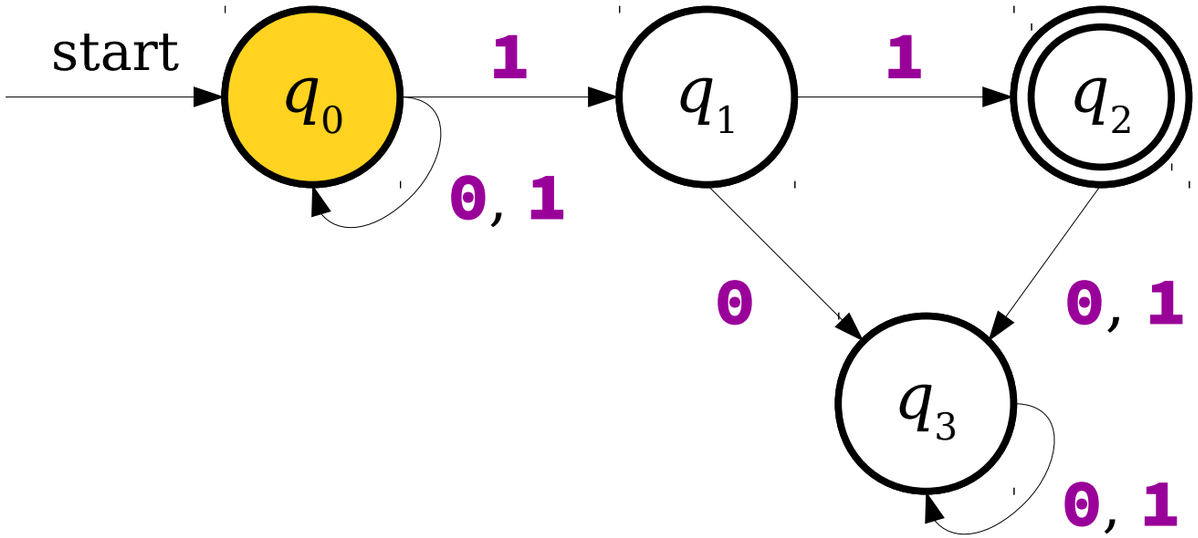

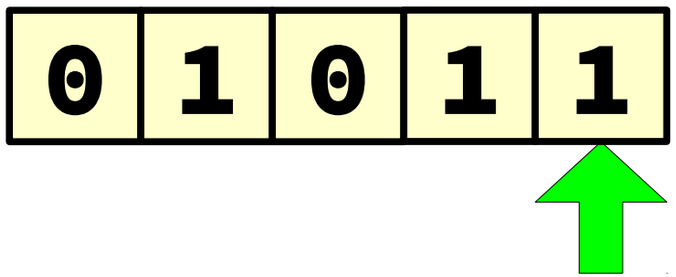

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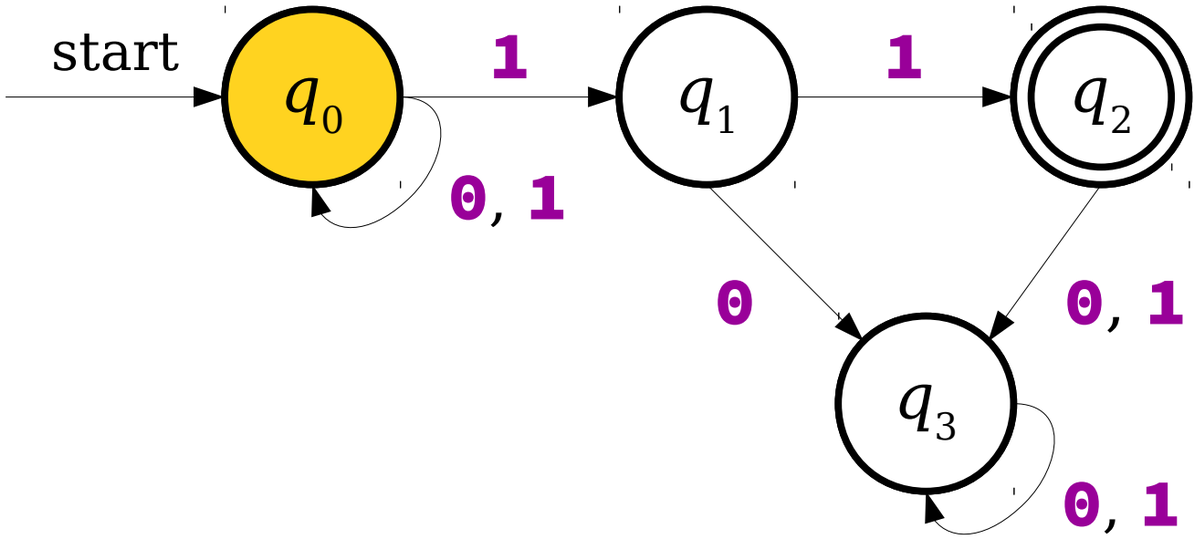
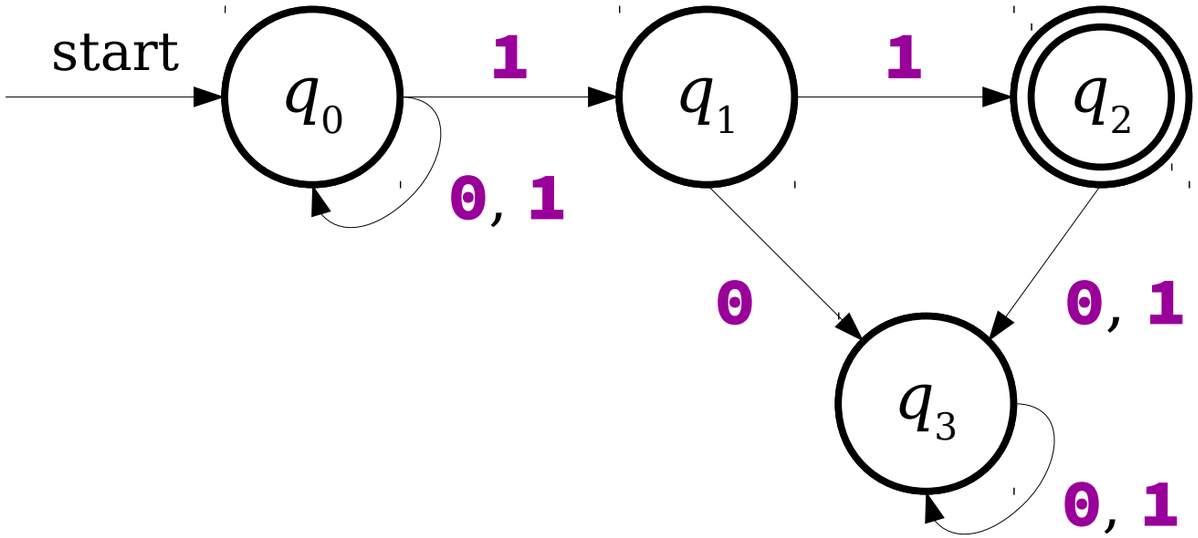


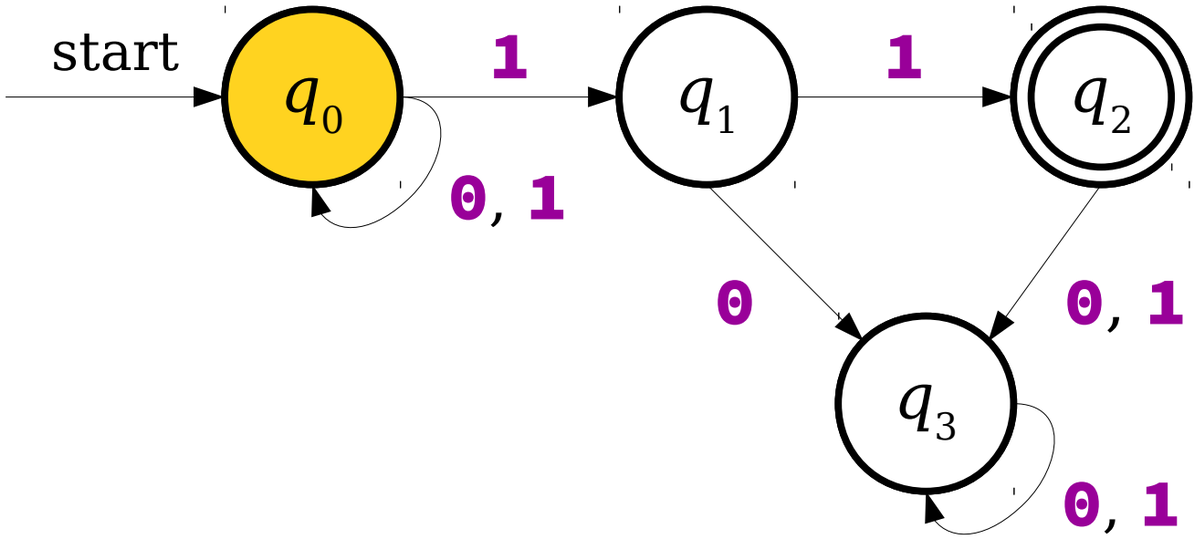
0	1	0	1	1

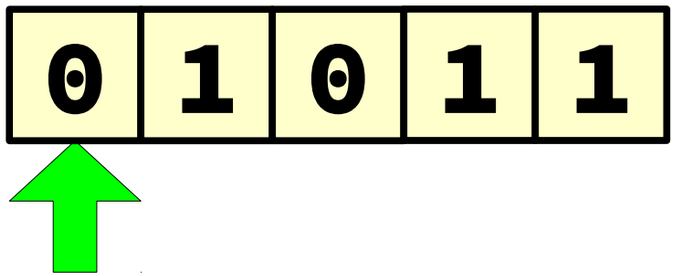
A Simple NFA



0	1	0	1	1

A Simple NFA





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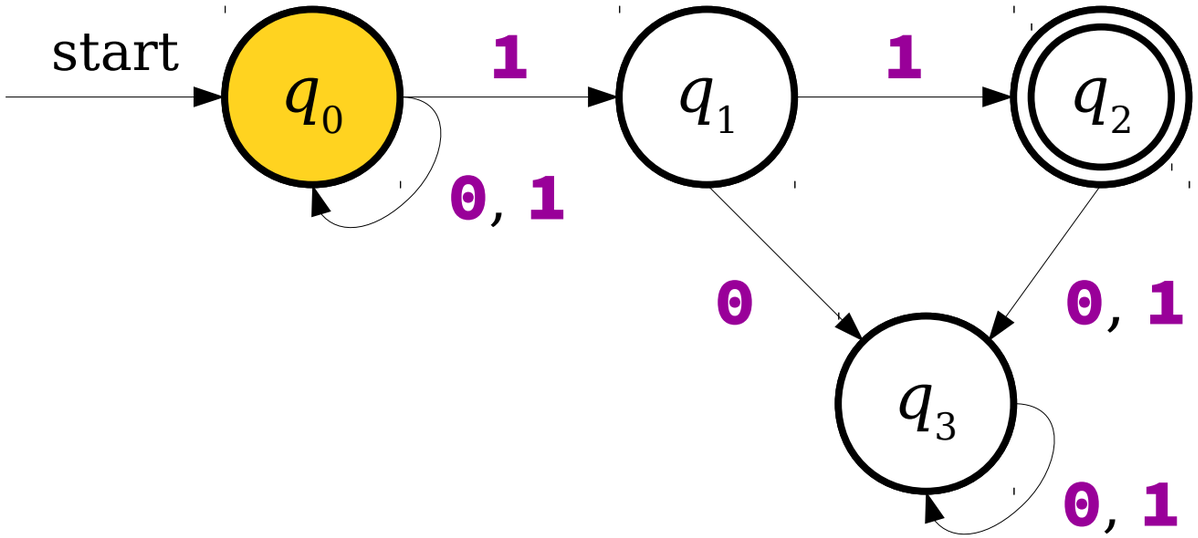

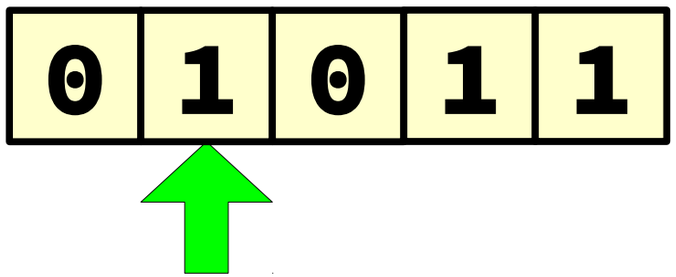

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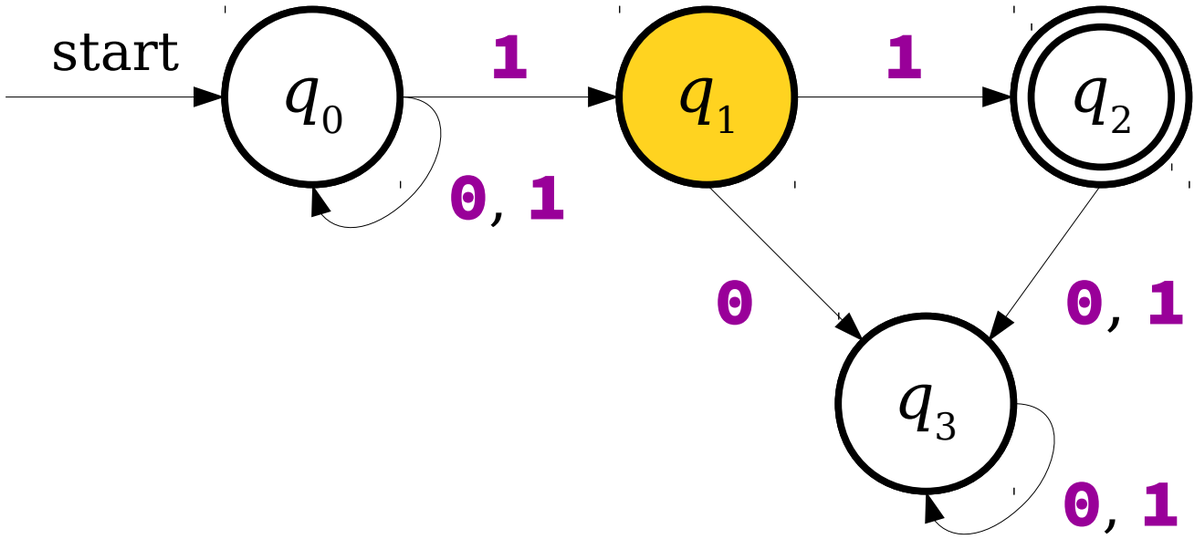

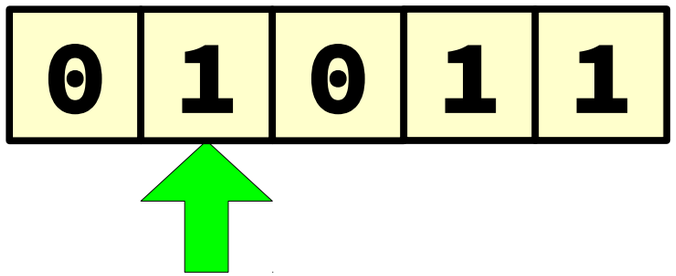

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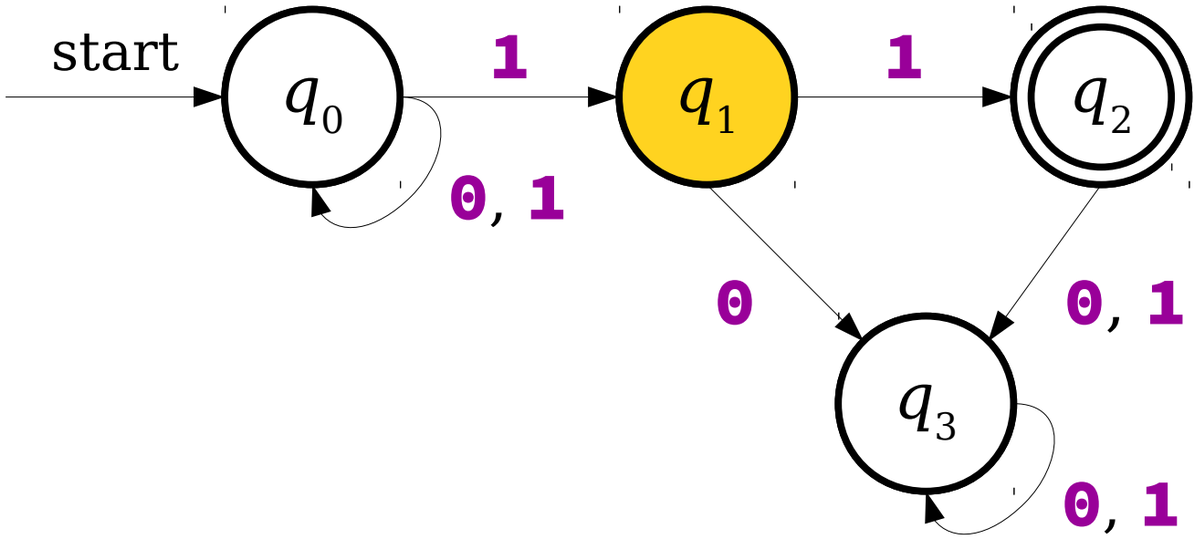

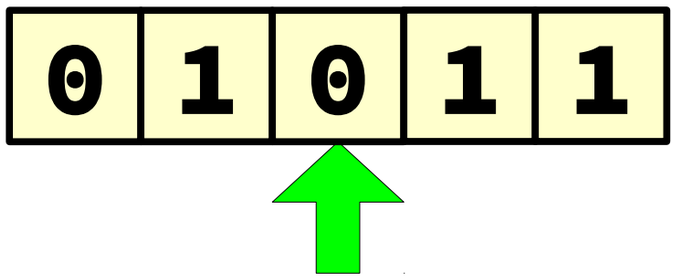

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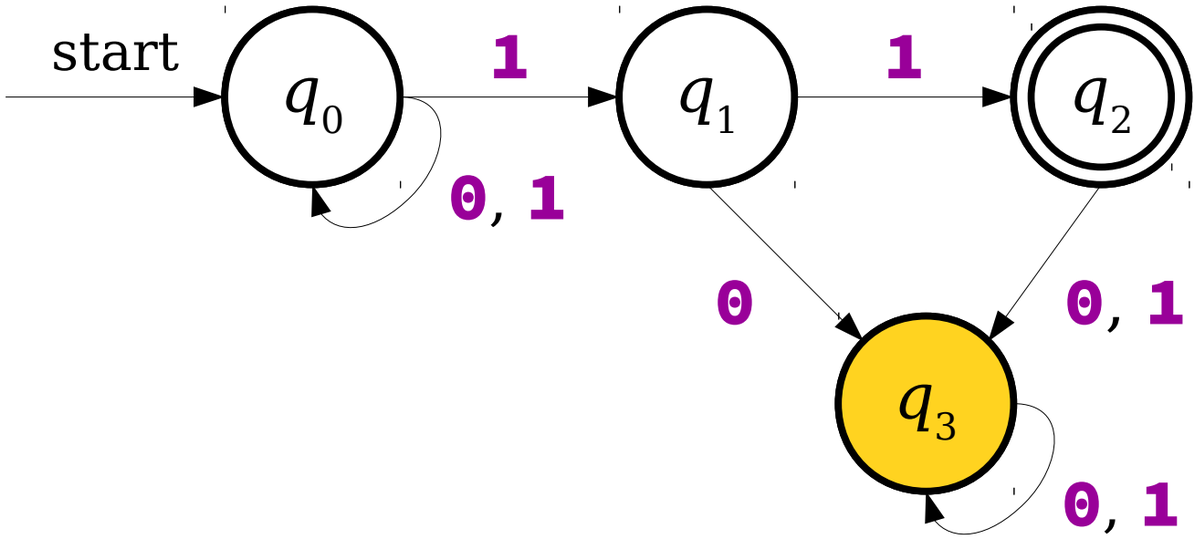

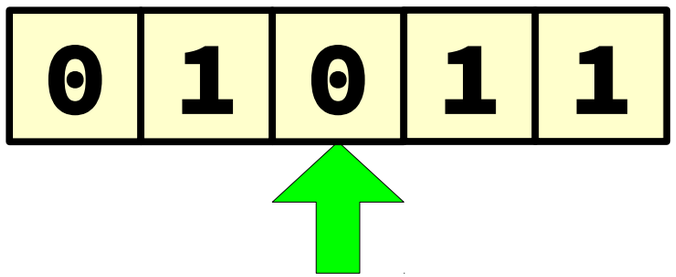

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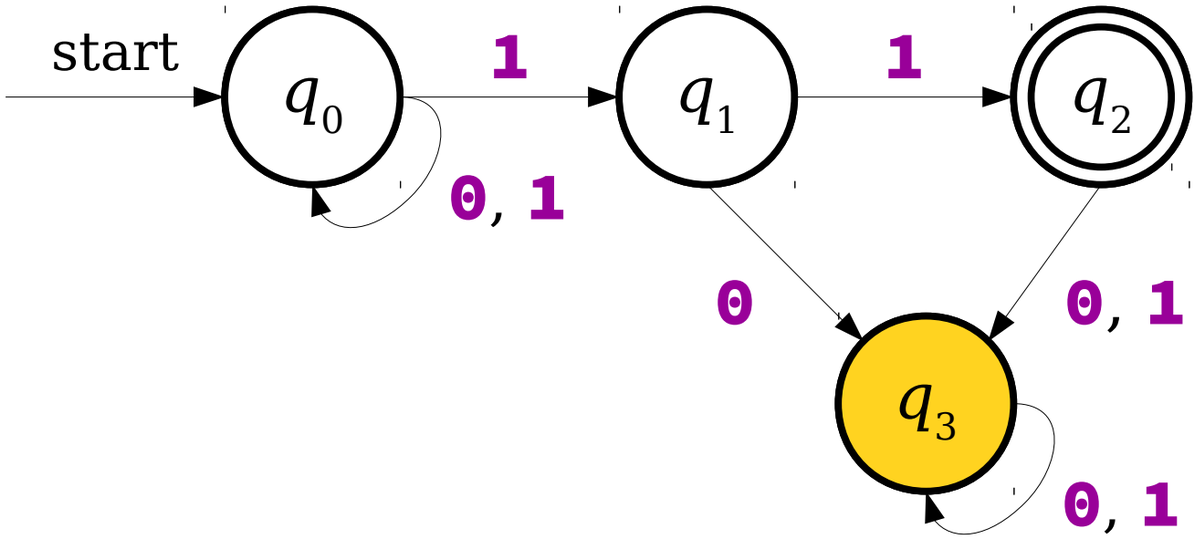

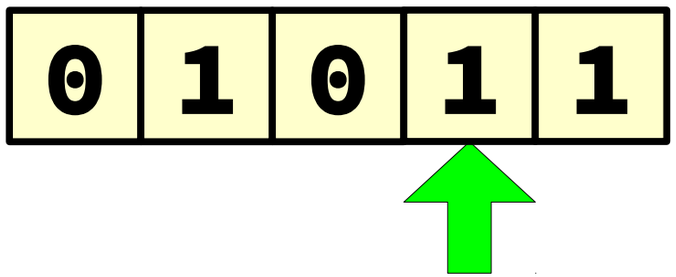

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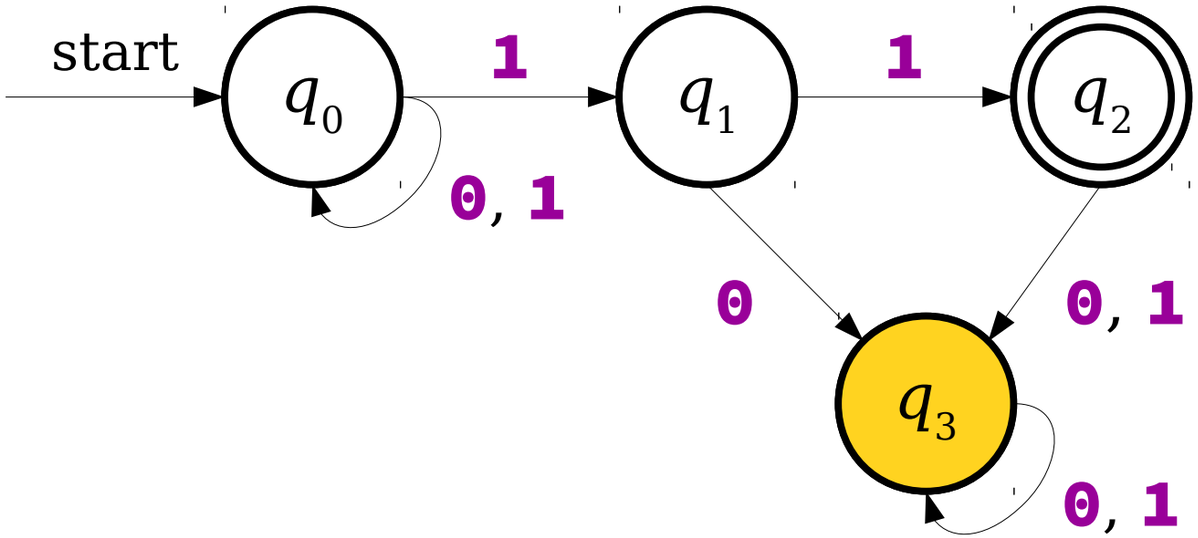

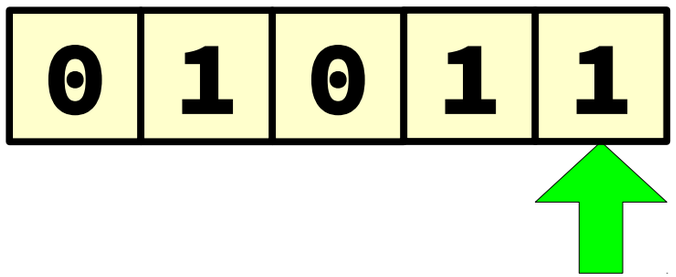

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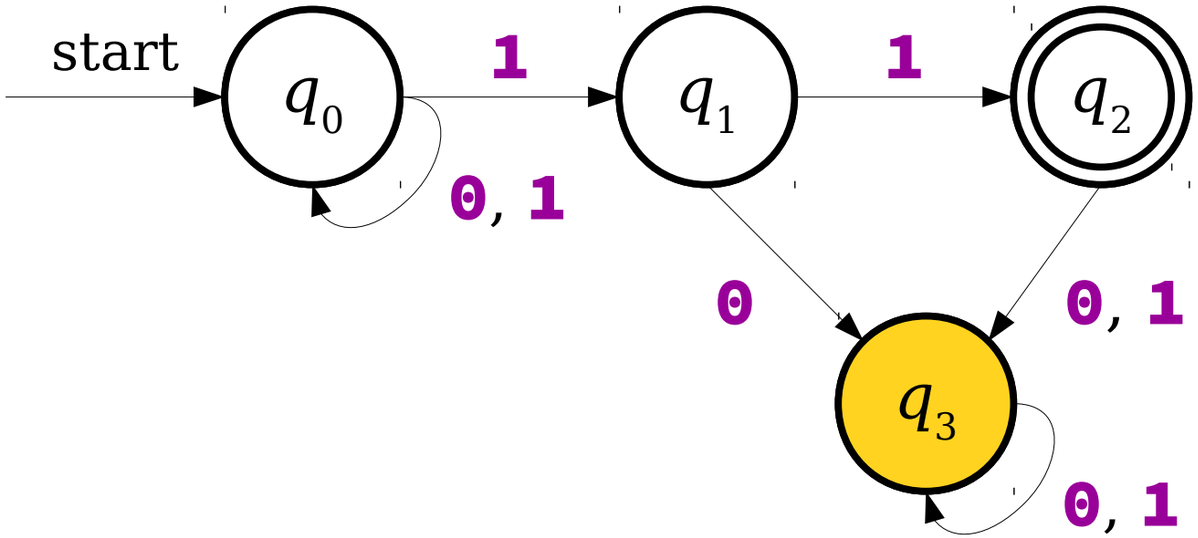

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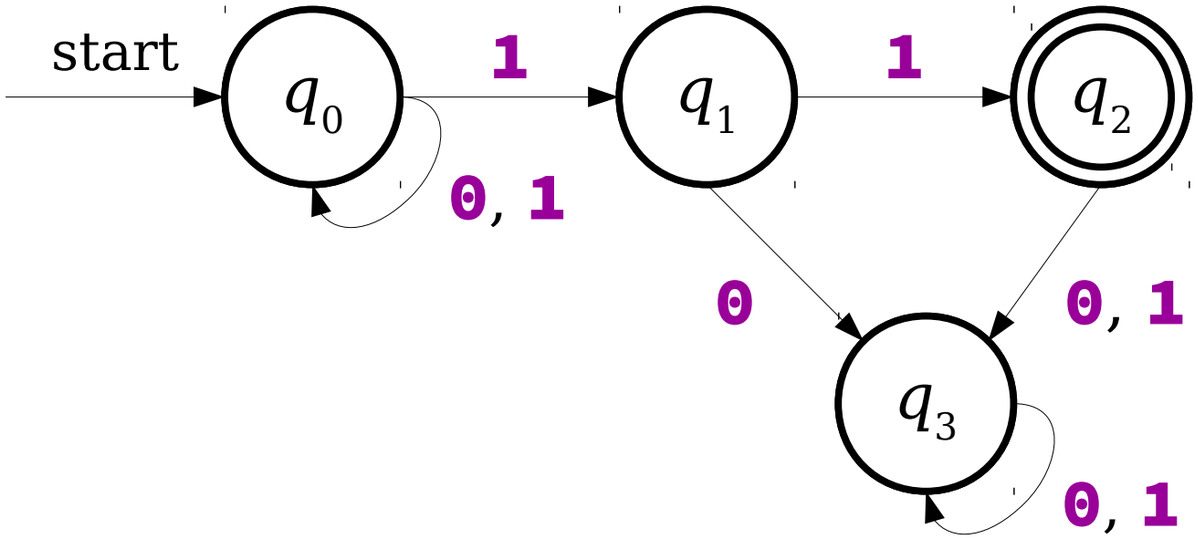

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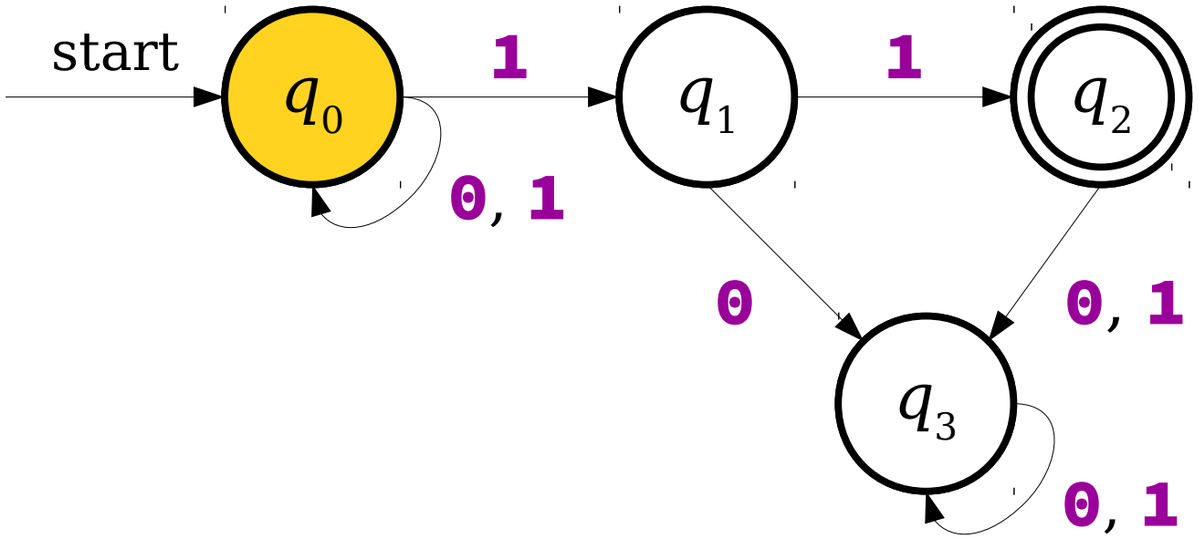

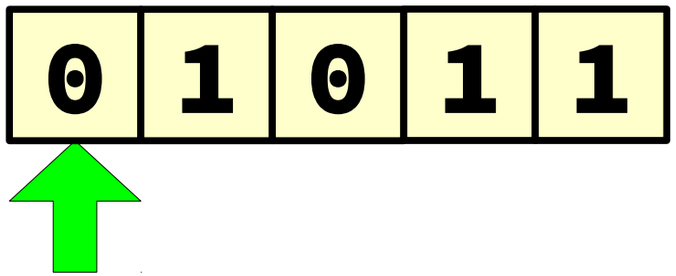

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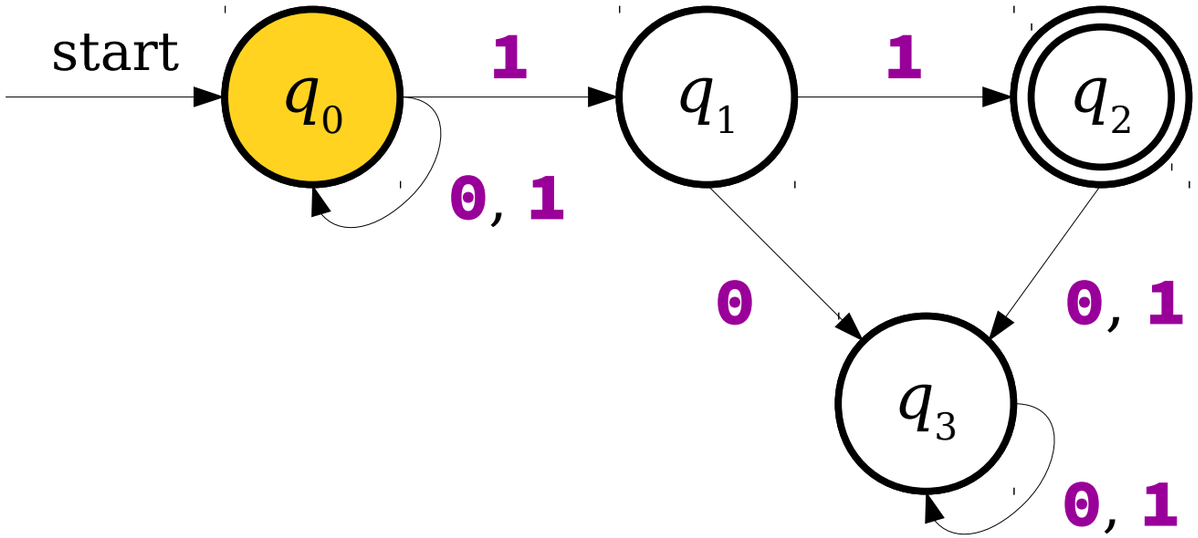

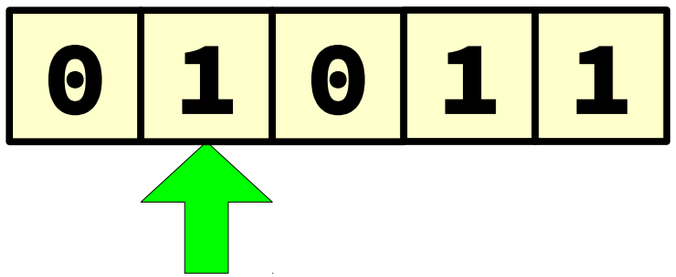

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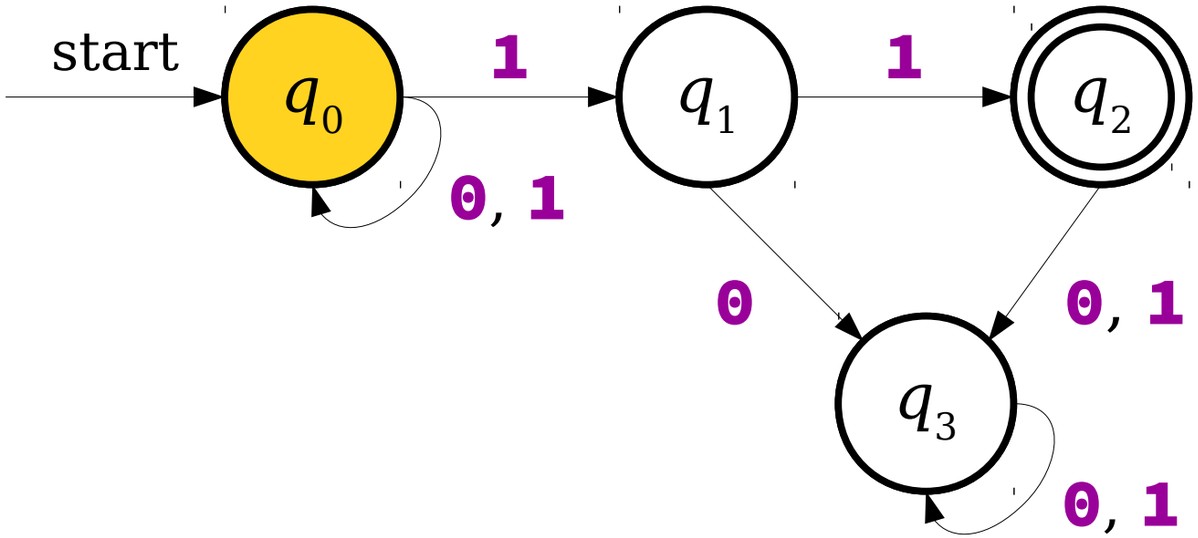

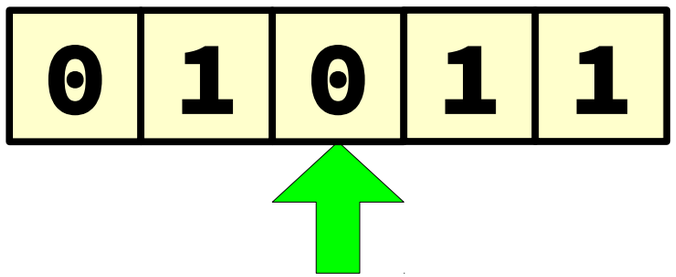

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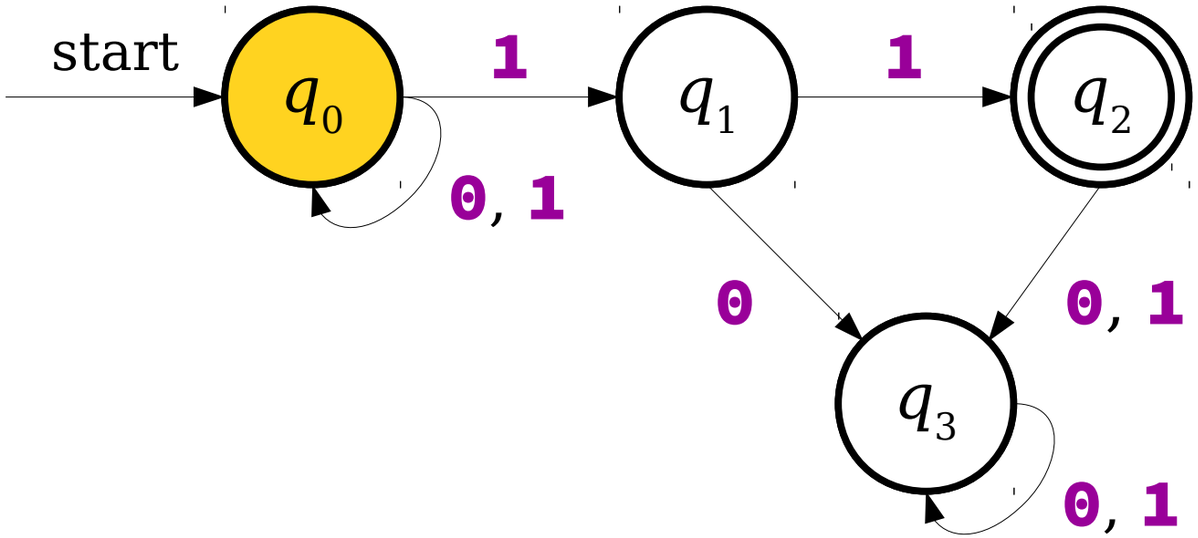

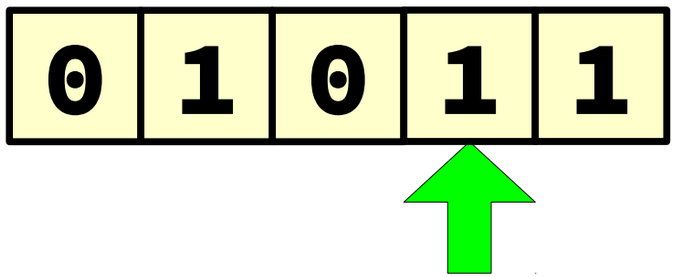

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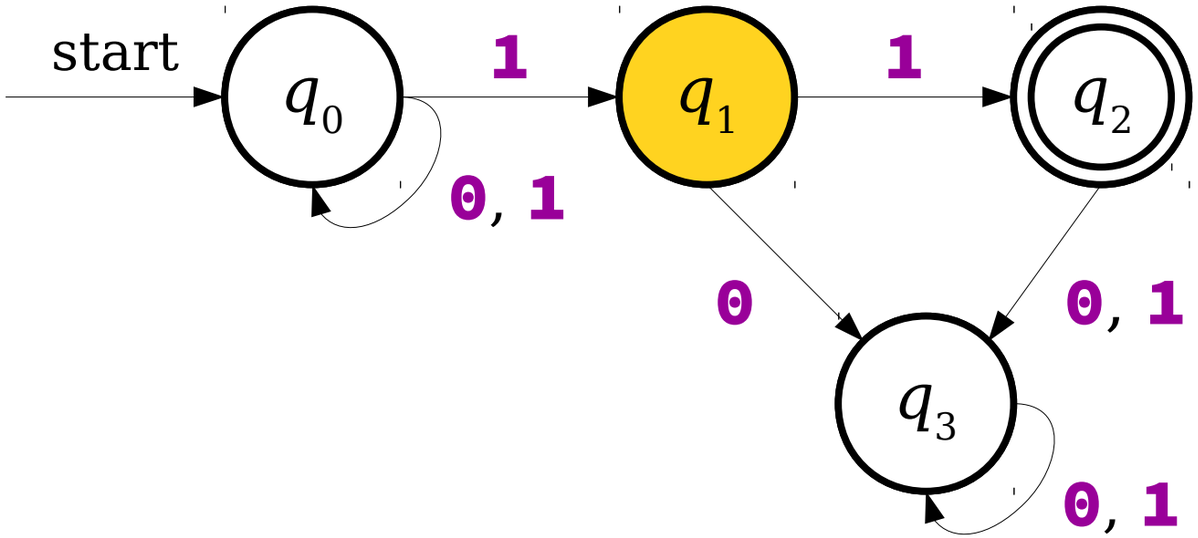

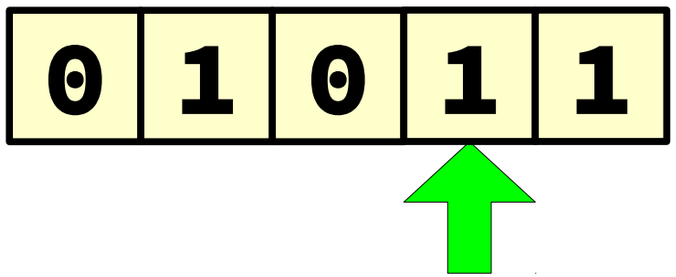

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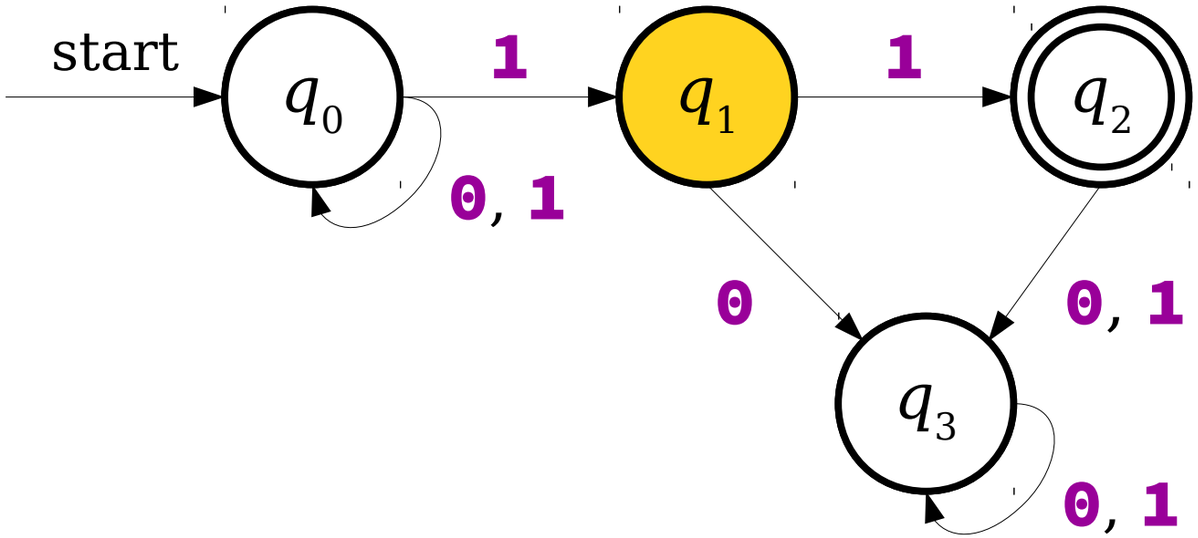

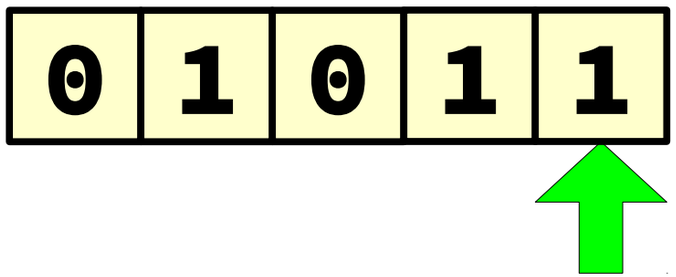

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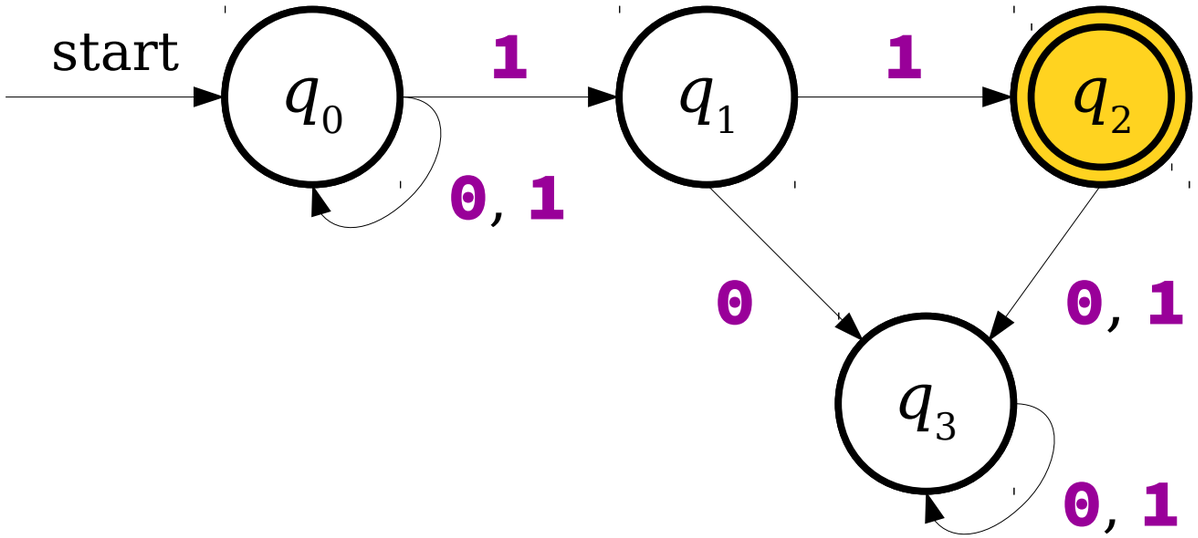

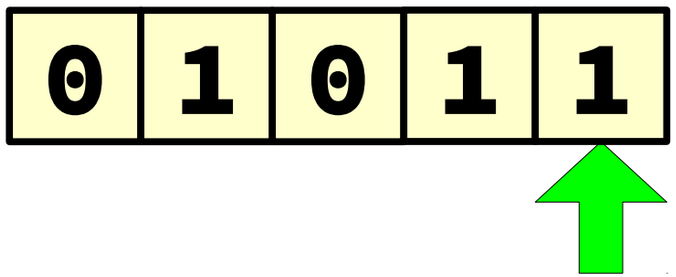

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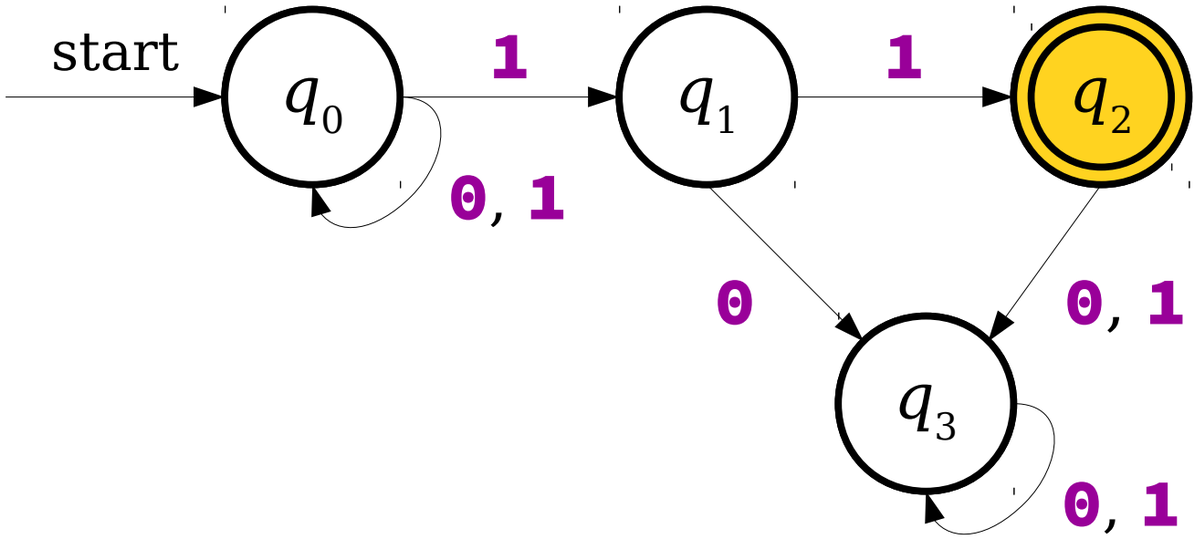

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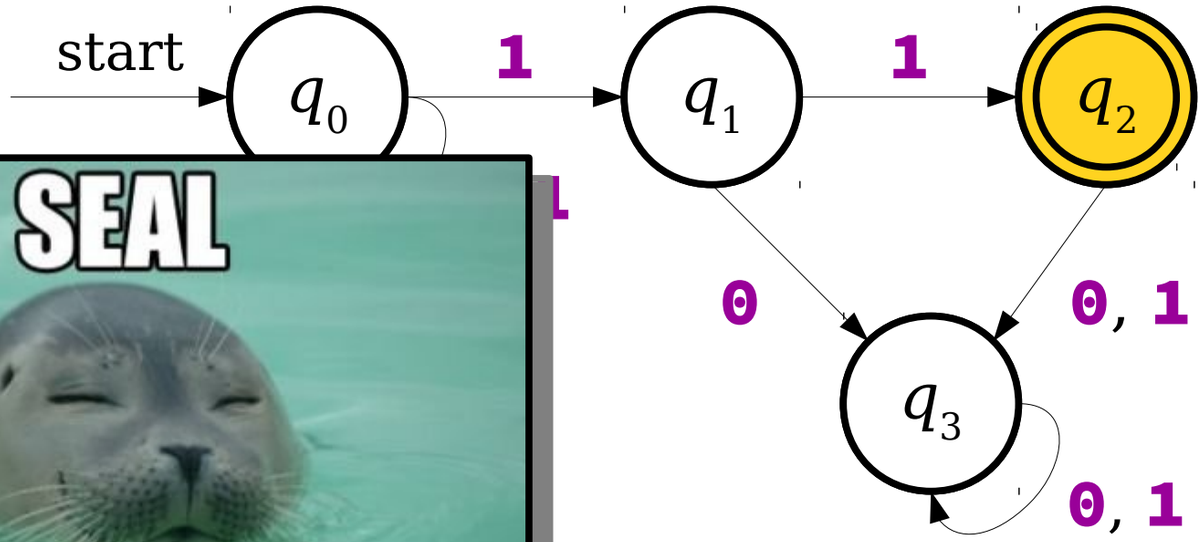

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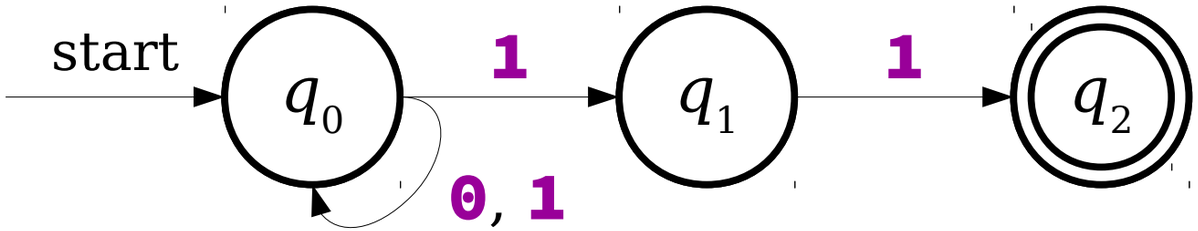

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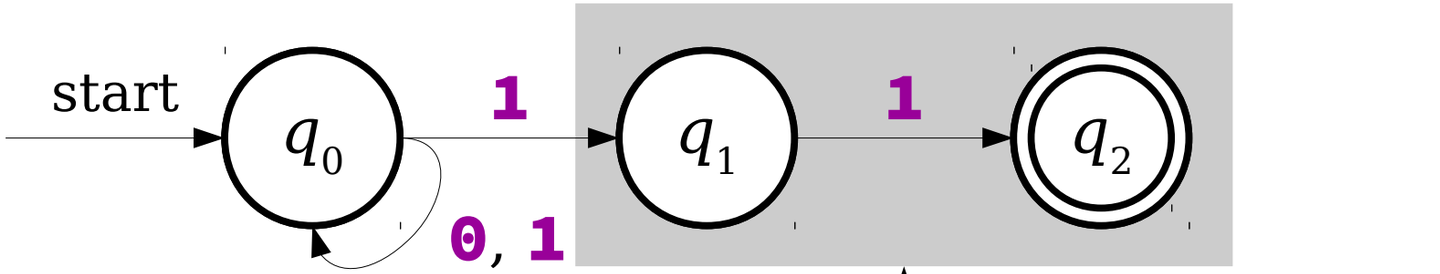
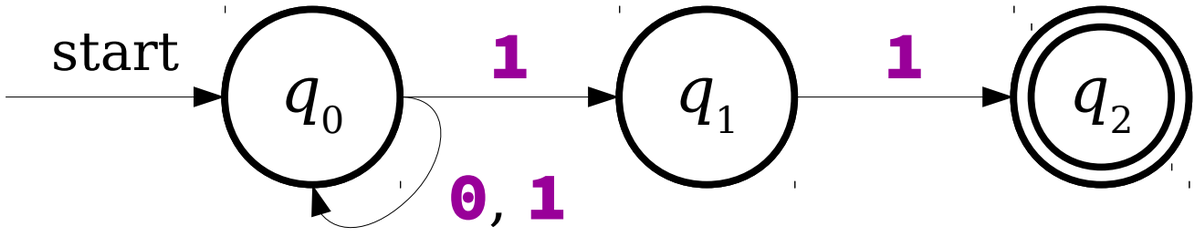


If a NFA needs to make a transition when no transition exists, the automaton **dies** and that particular path does not accept.

A More Complex NFA



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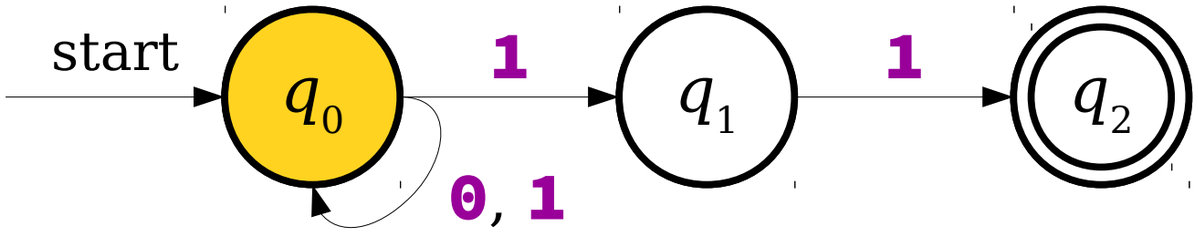

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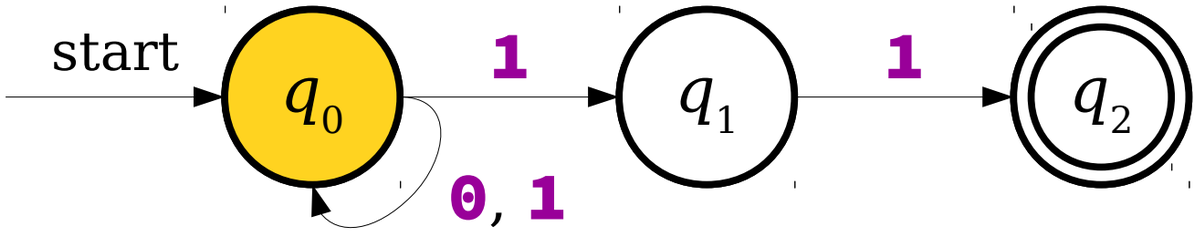

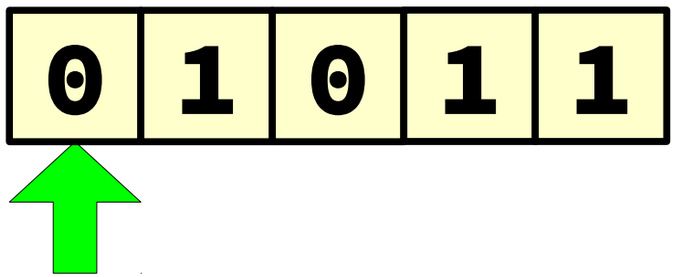

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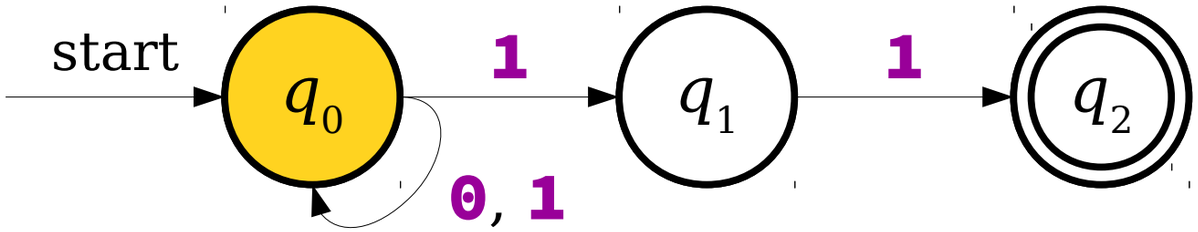

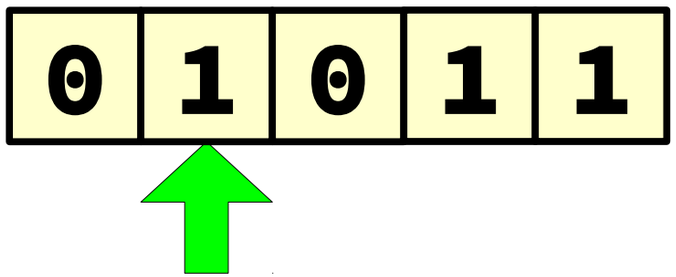

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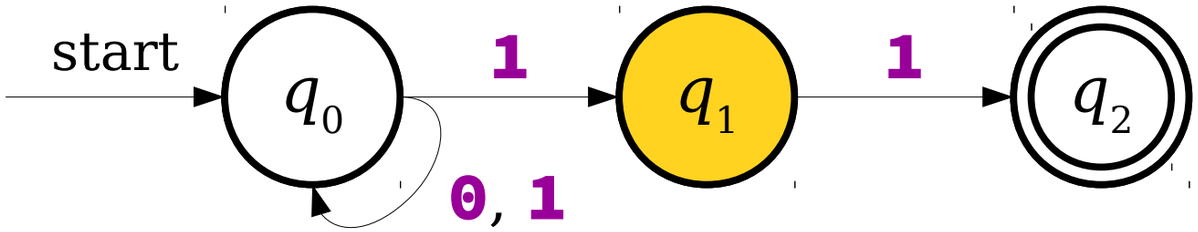

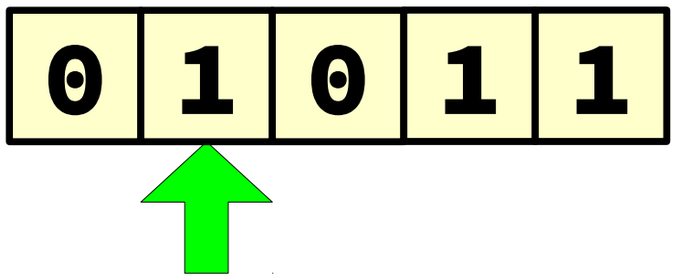

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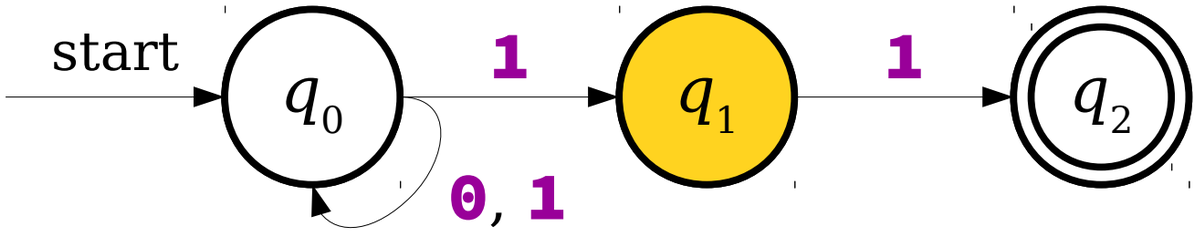

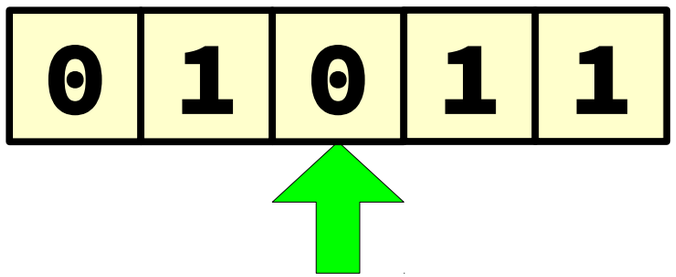

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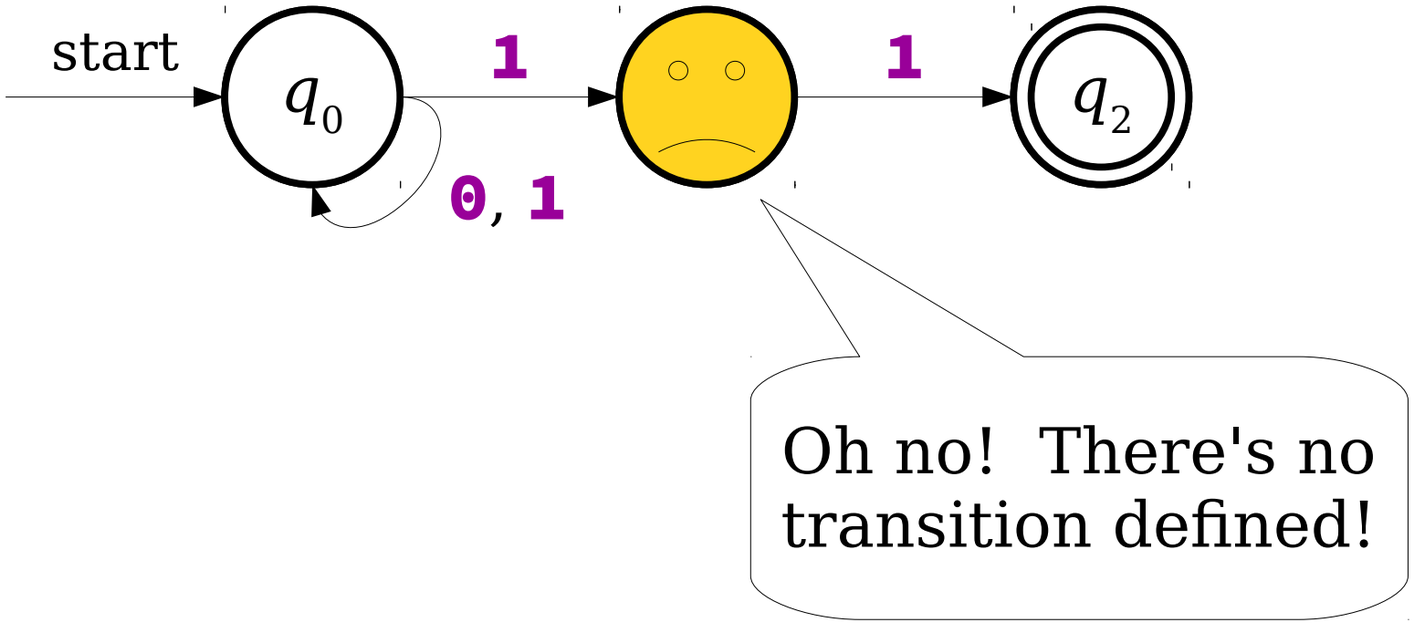

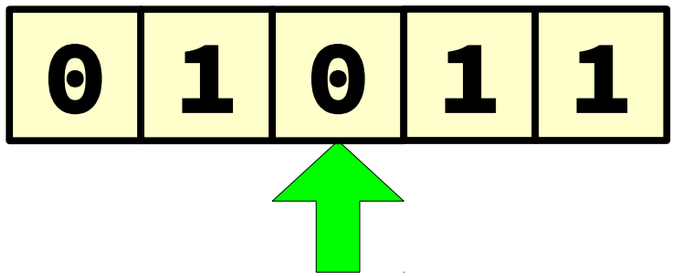

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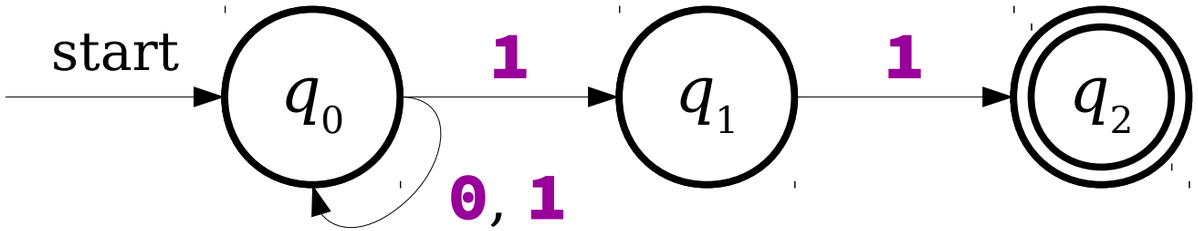

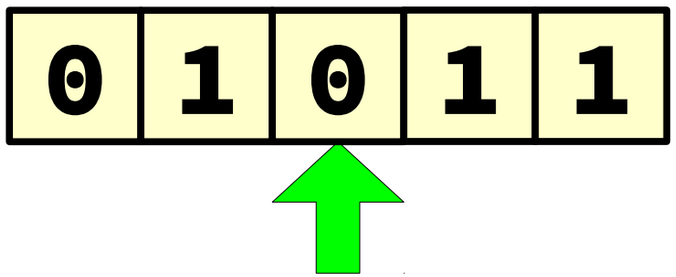

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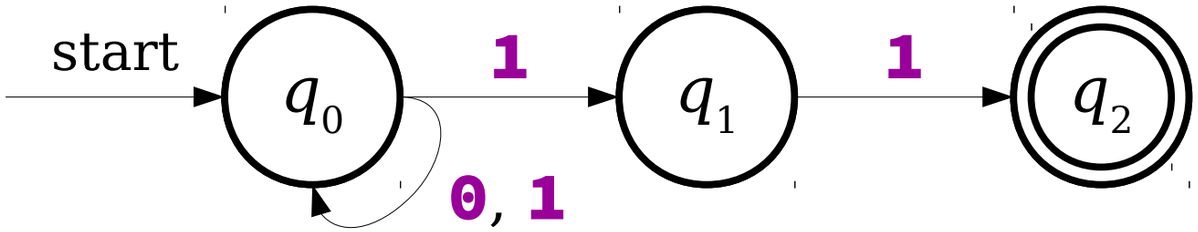

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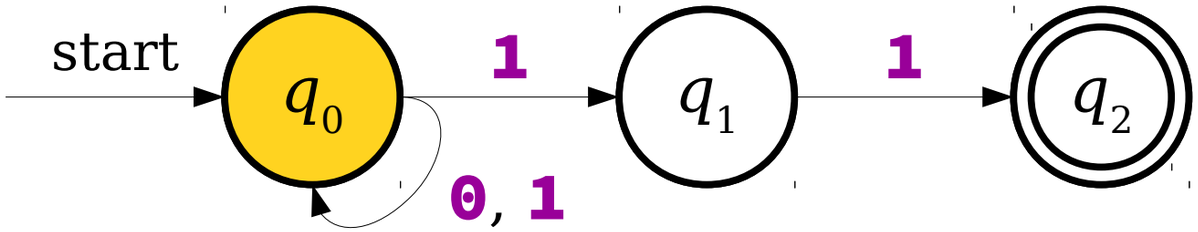

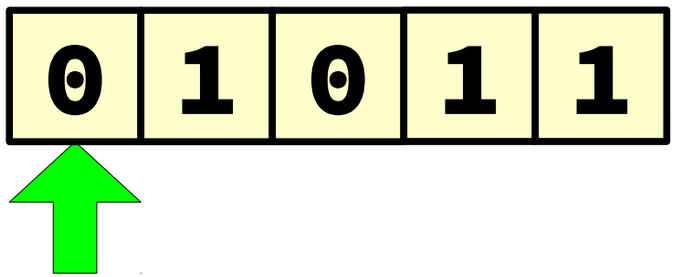

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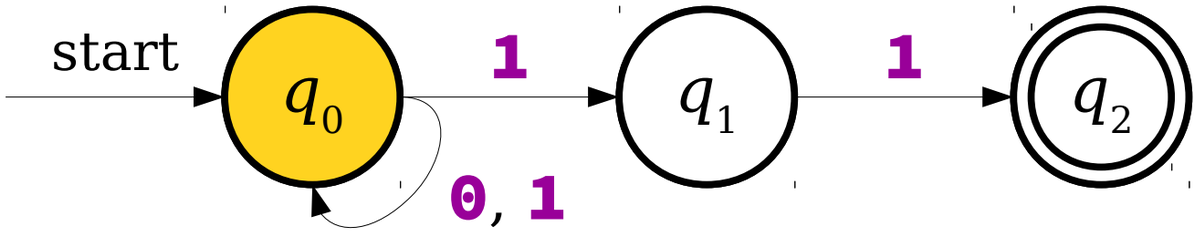

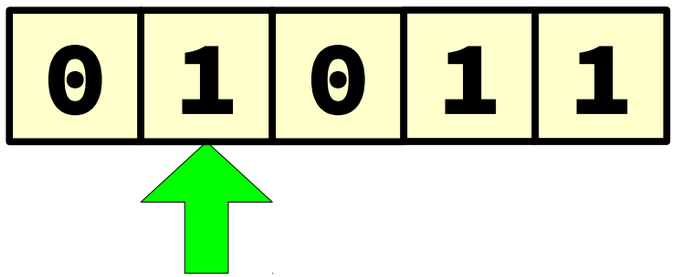

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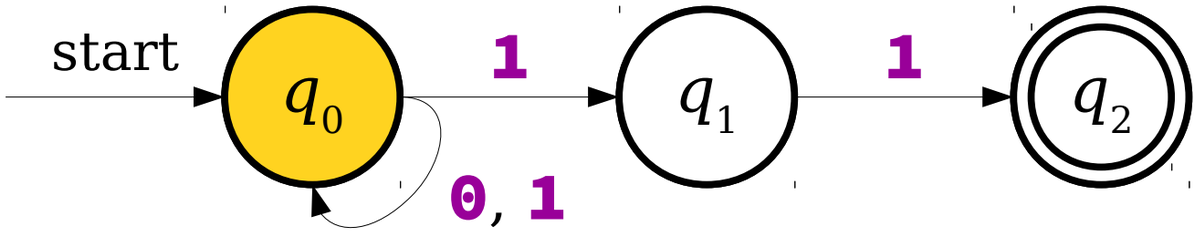

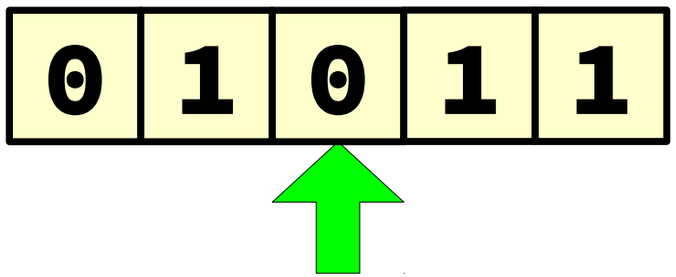

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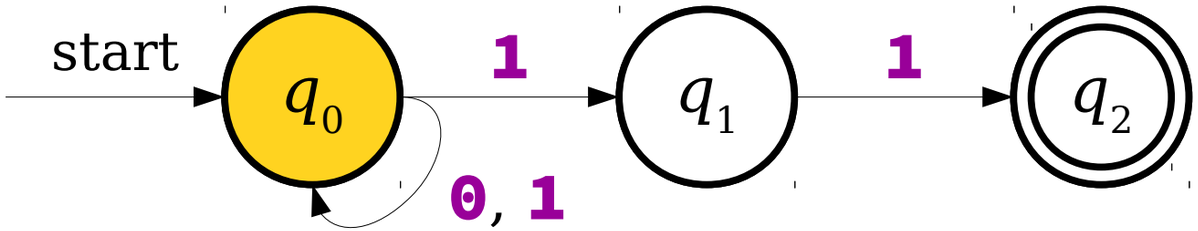

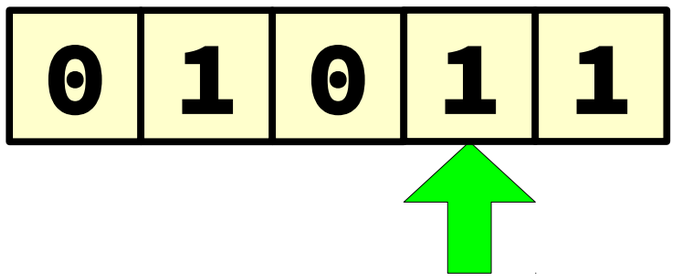

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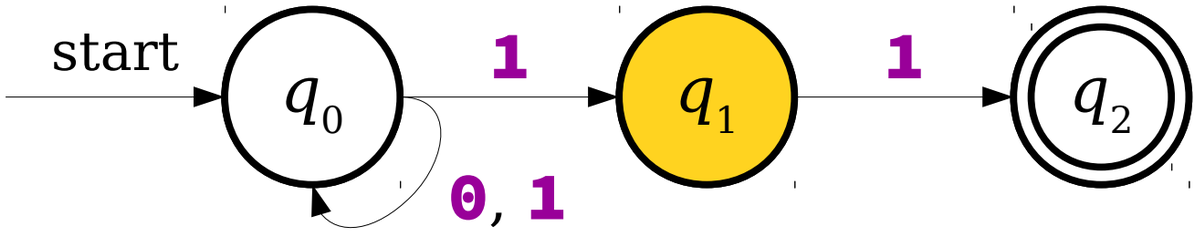

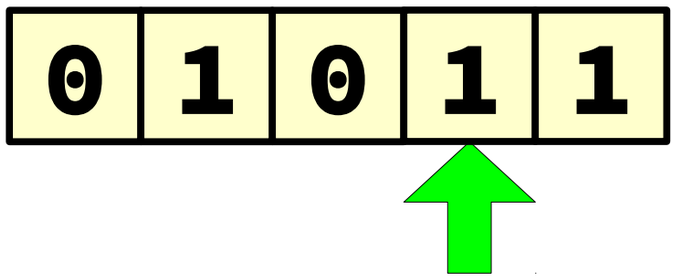

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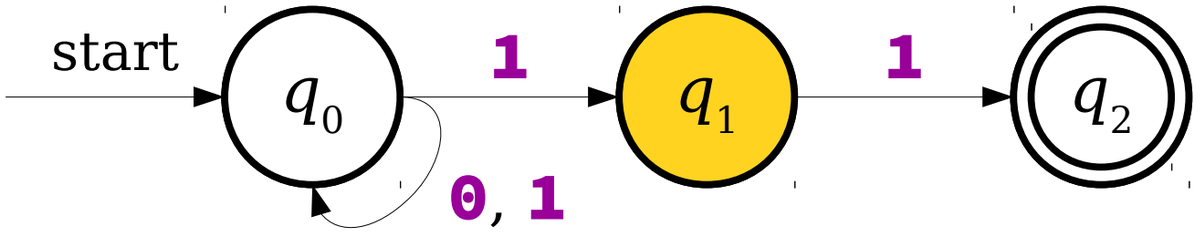

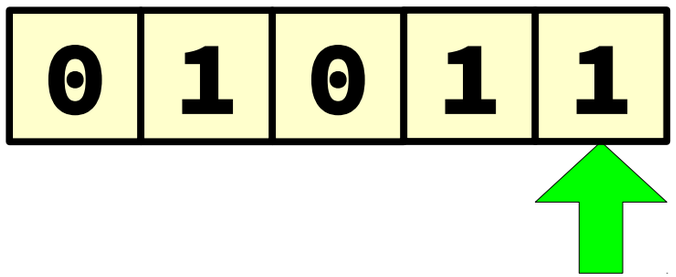

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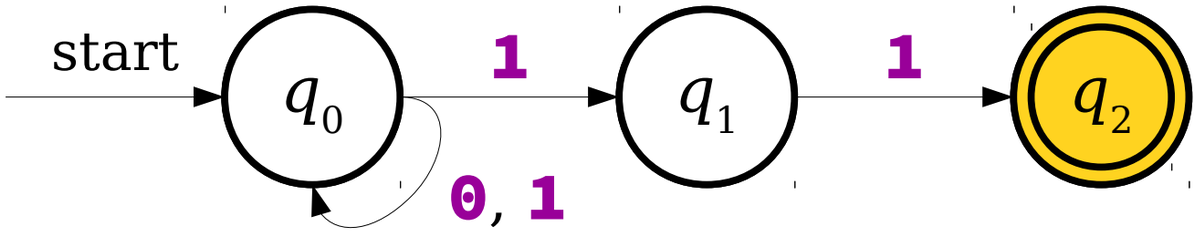

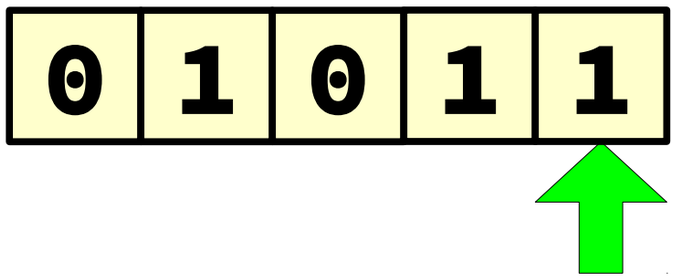

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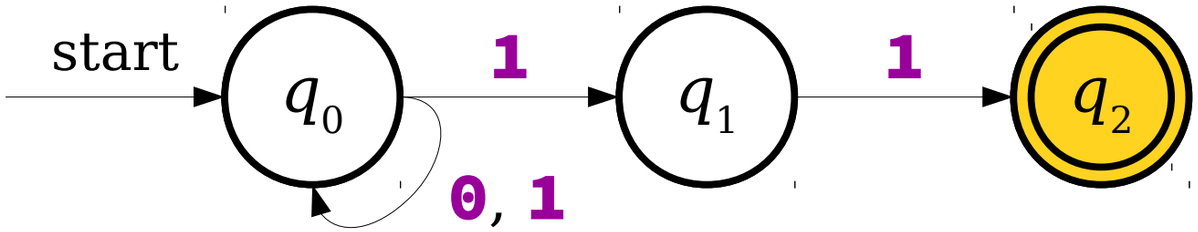

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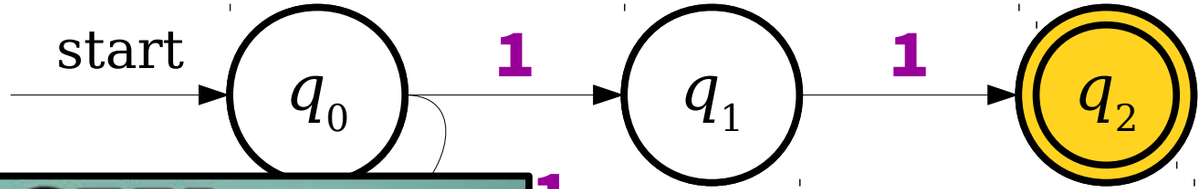
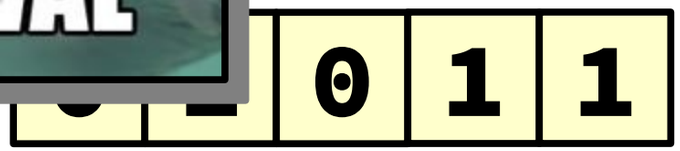
A More Complex NFA





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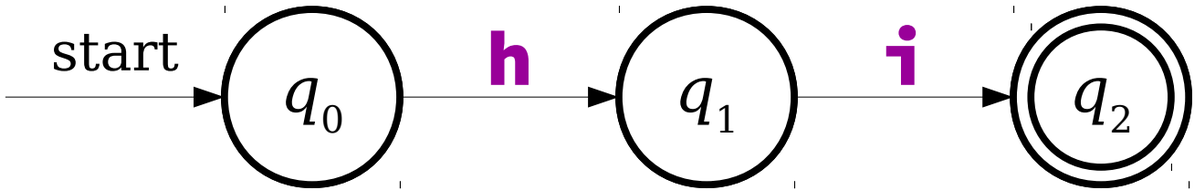

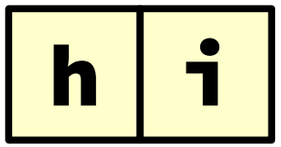

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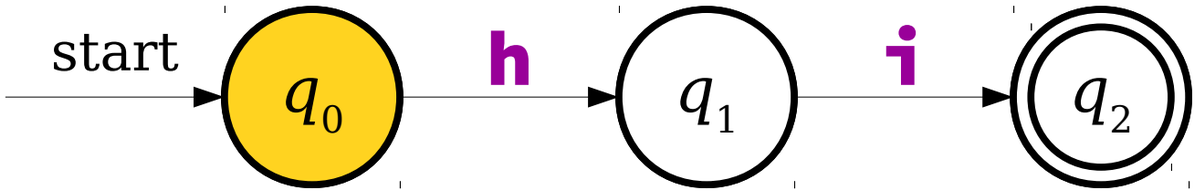

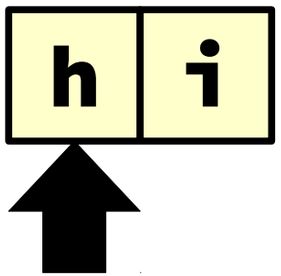

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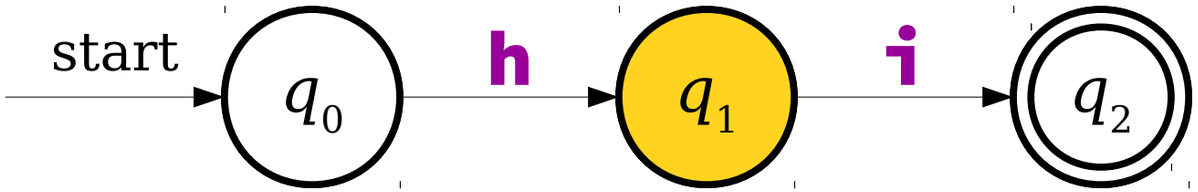

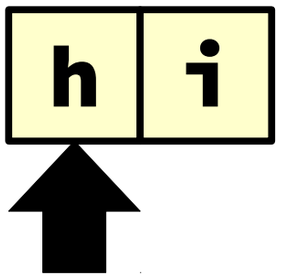

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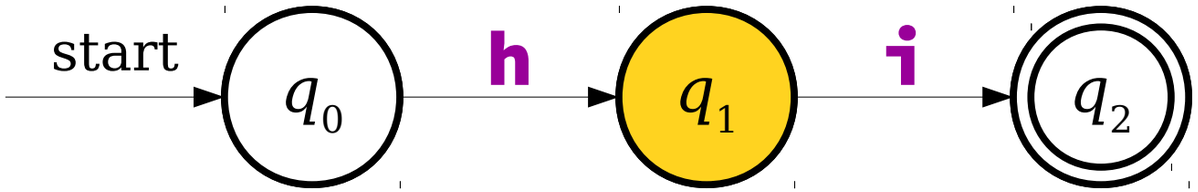

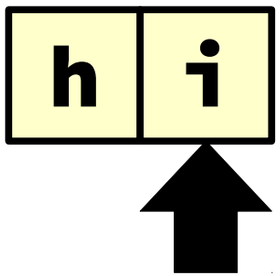

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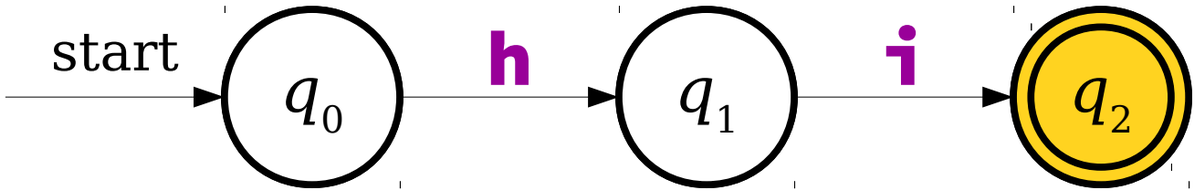

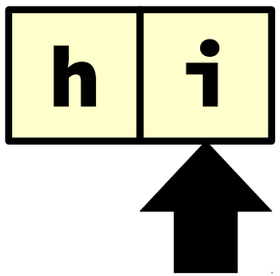

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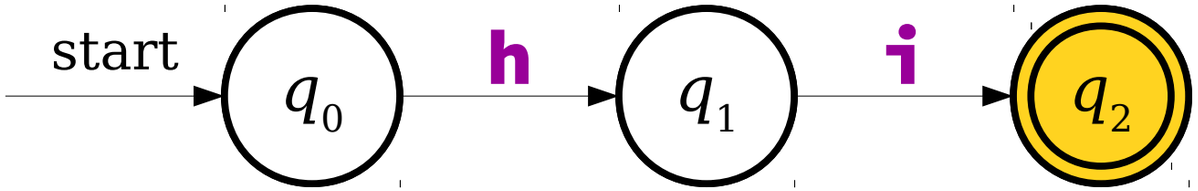

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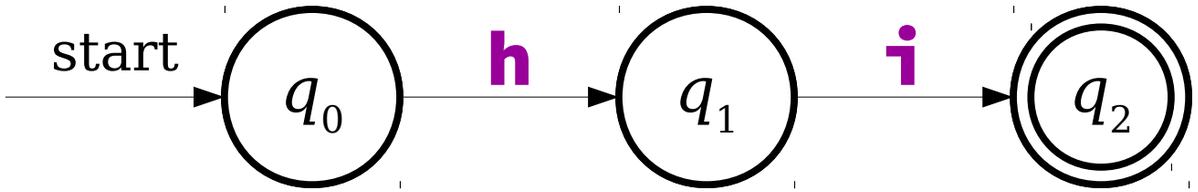
Tragedy in Paradise



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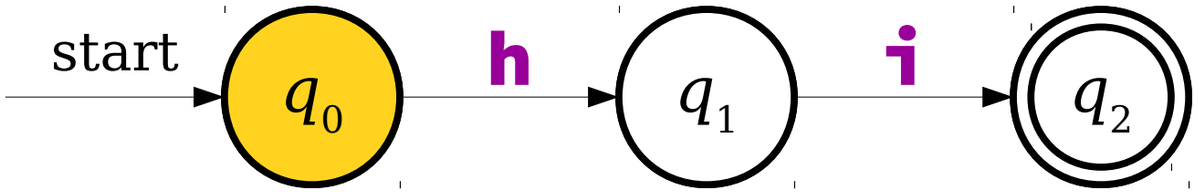

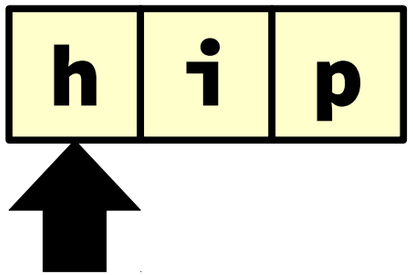

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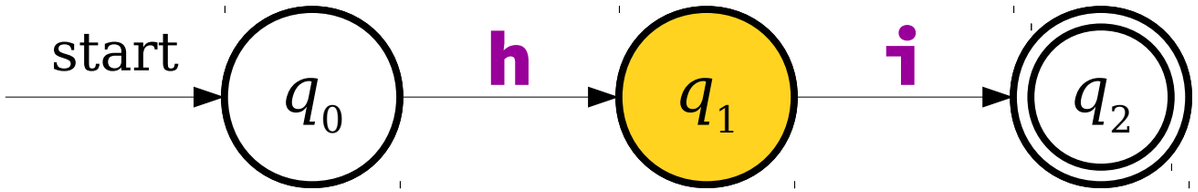

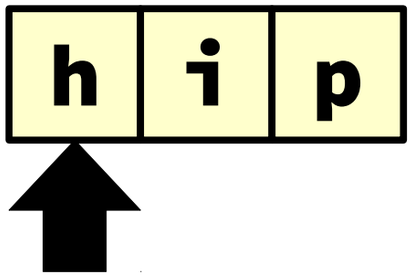

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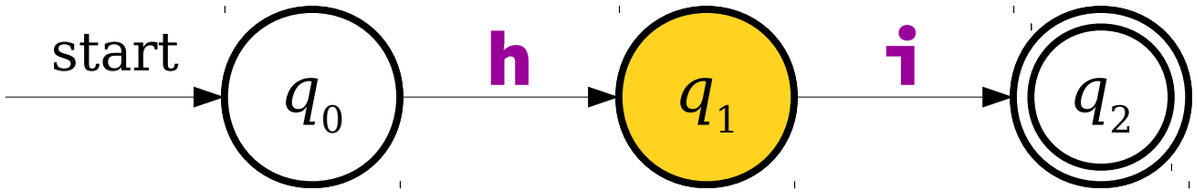

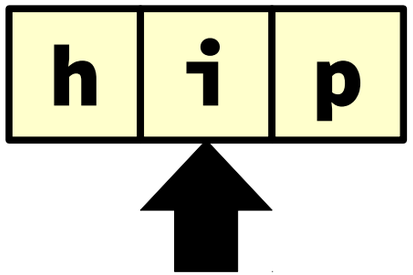

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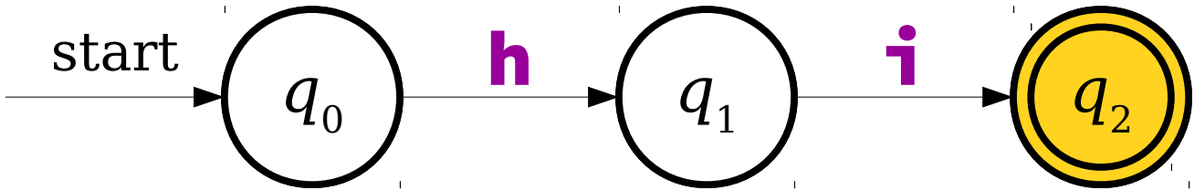

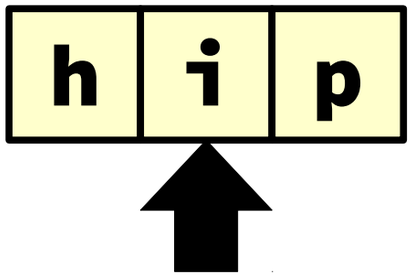

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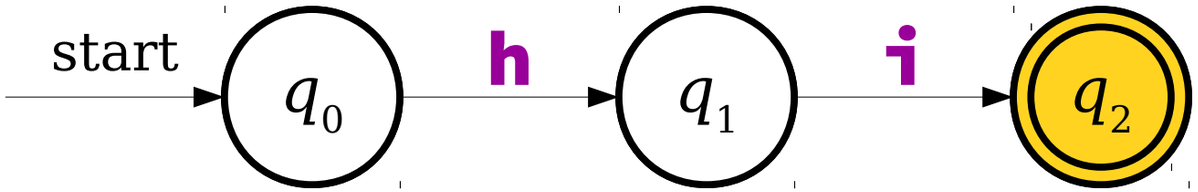

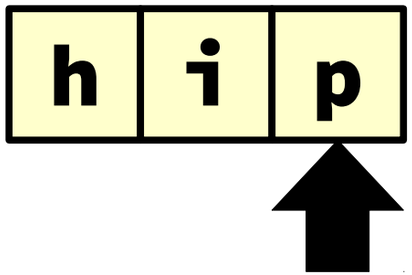

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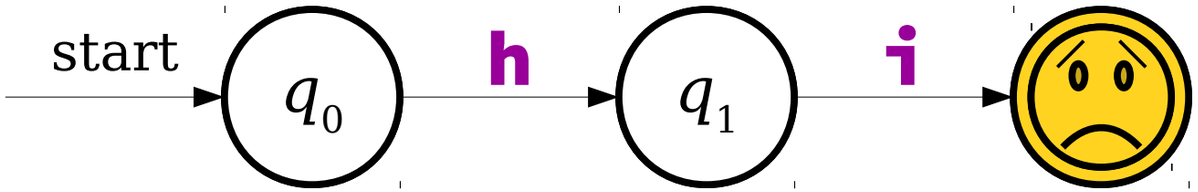

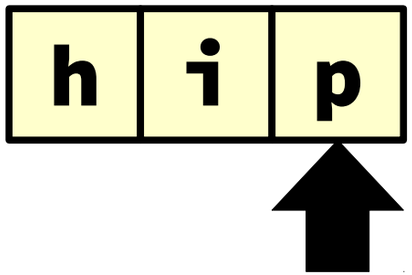

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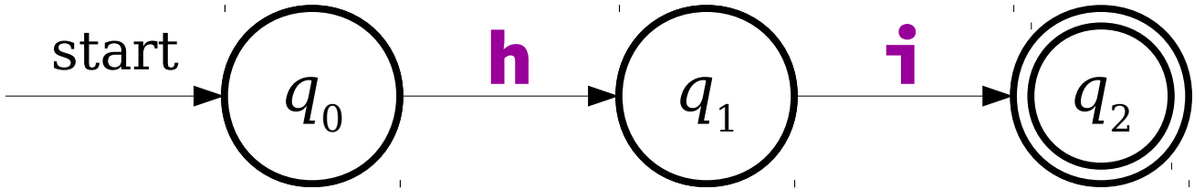

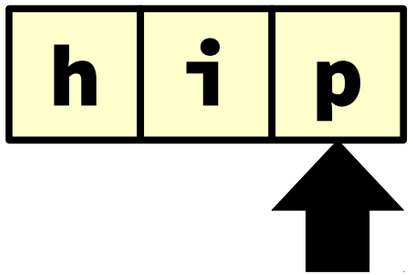

ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
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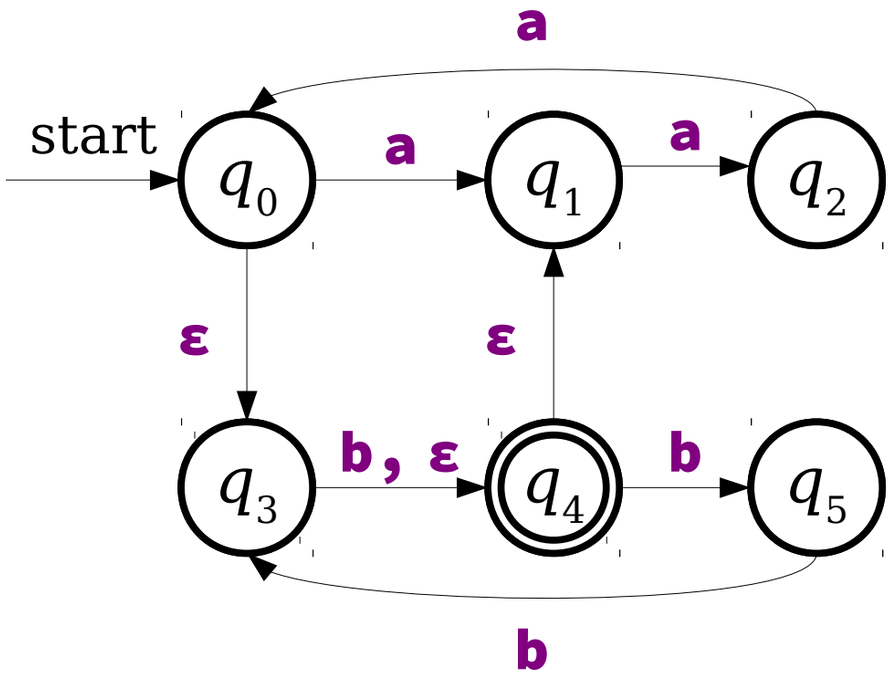

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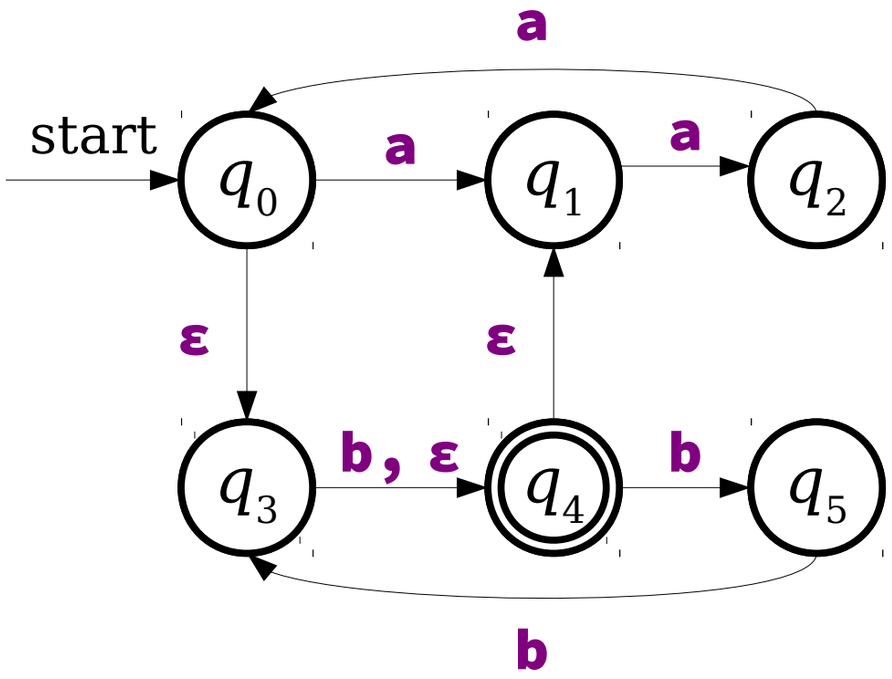

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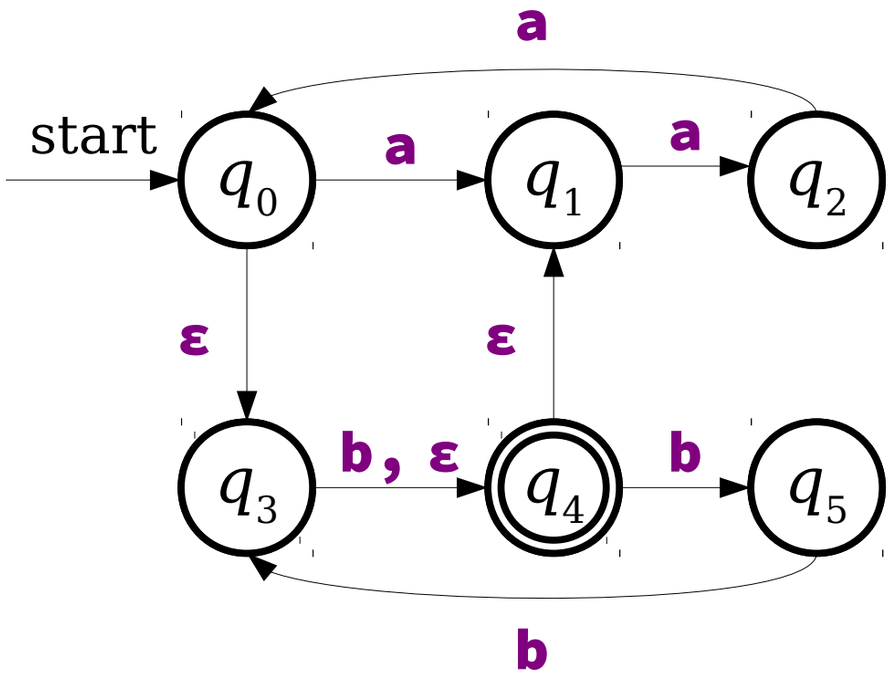

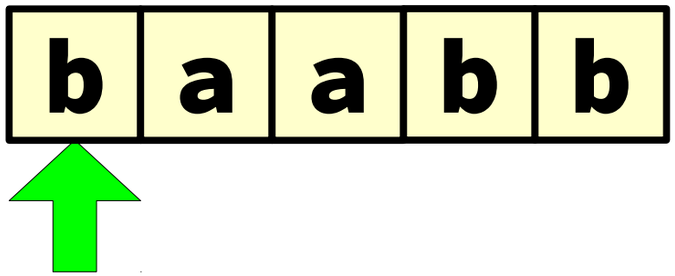

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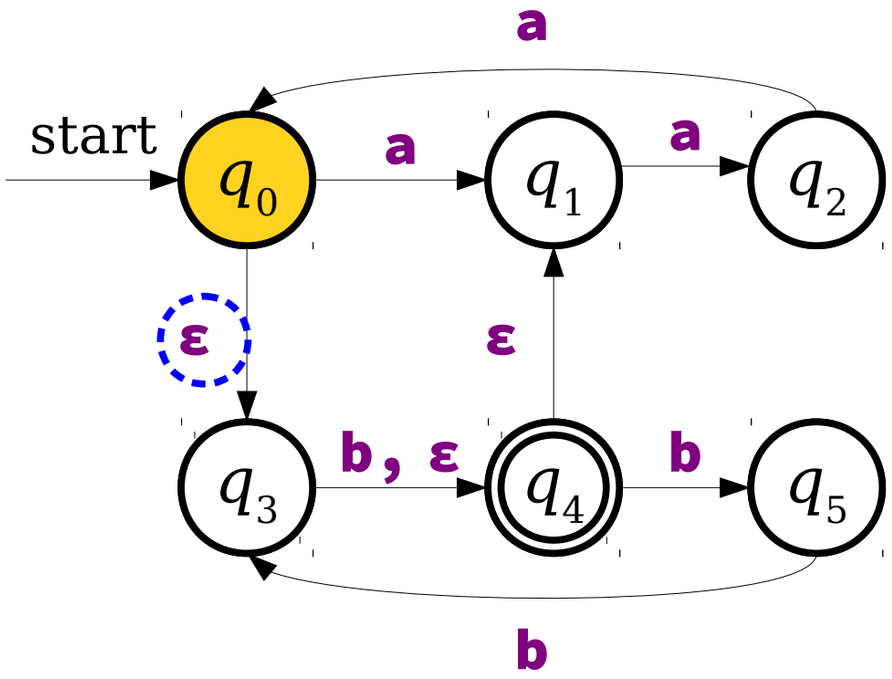

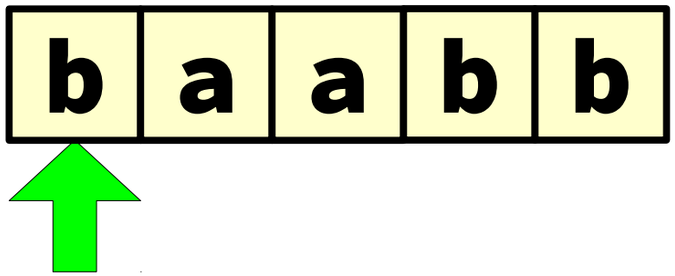

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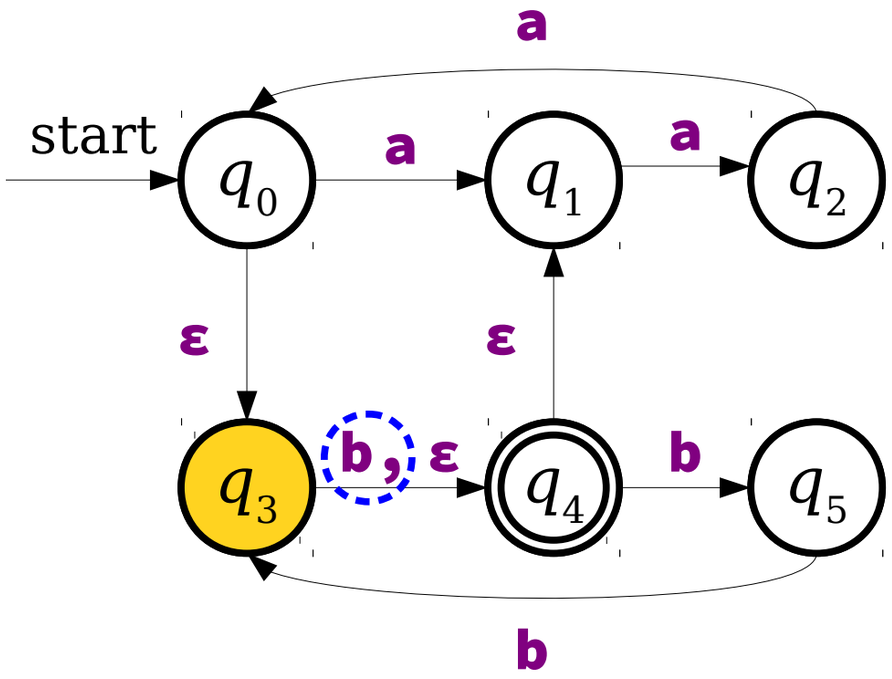

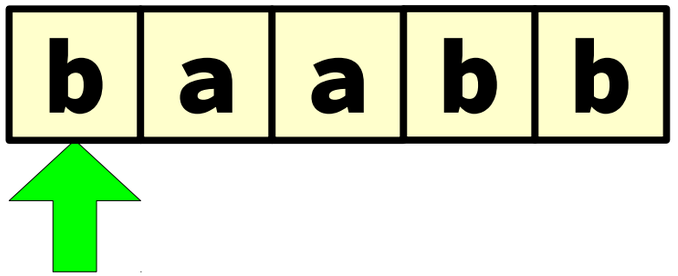

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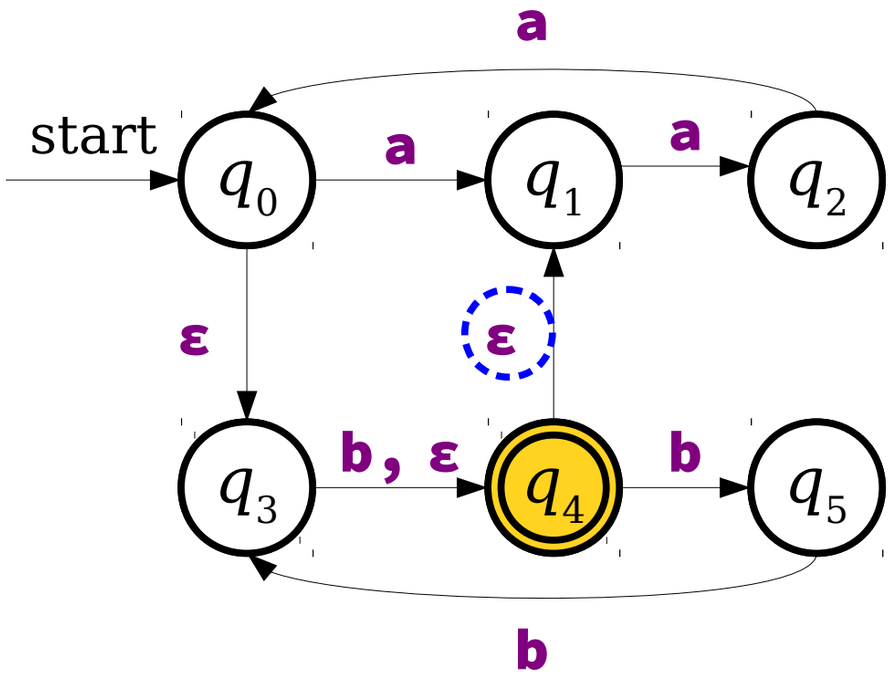

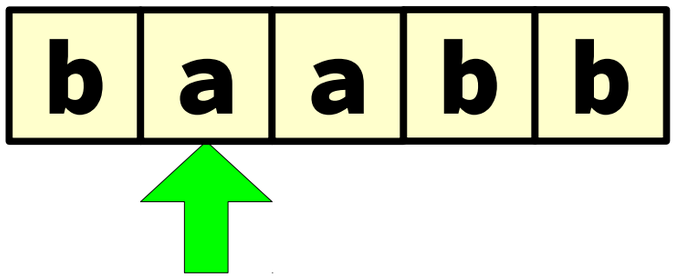

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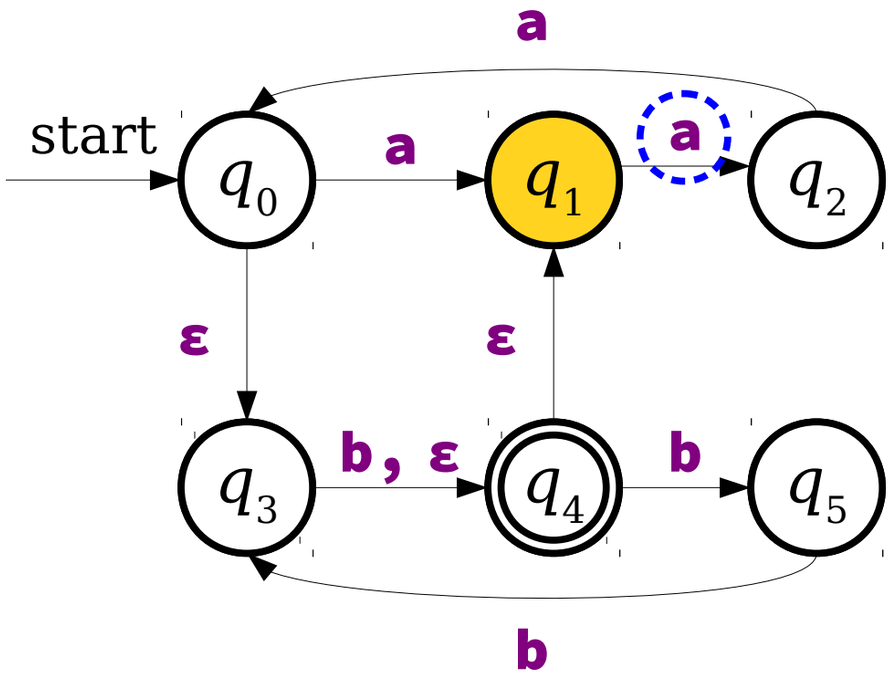

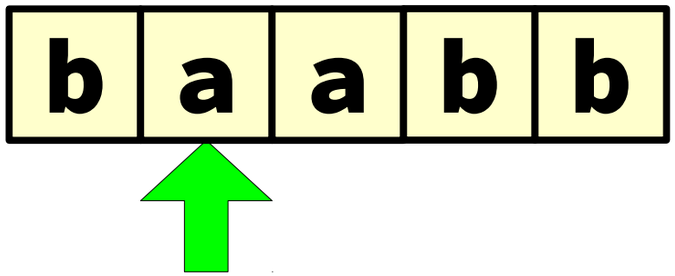

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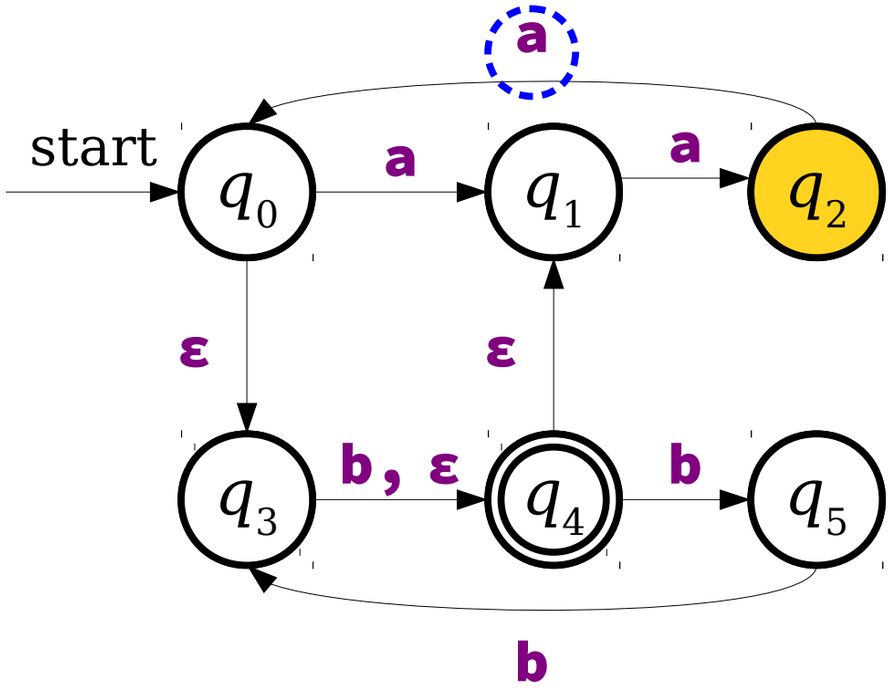

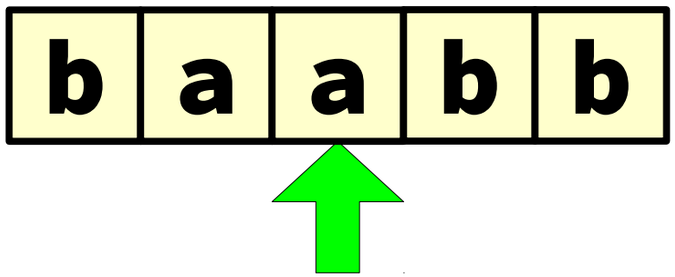

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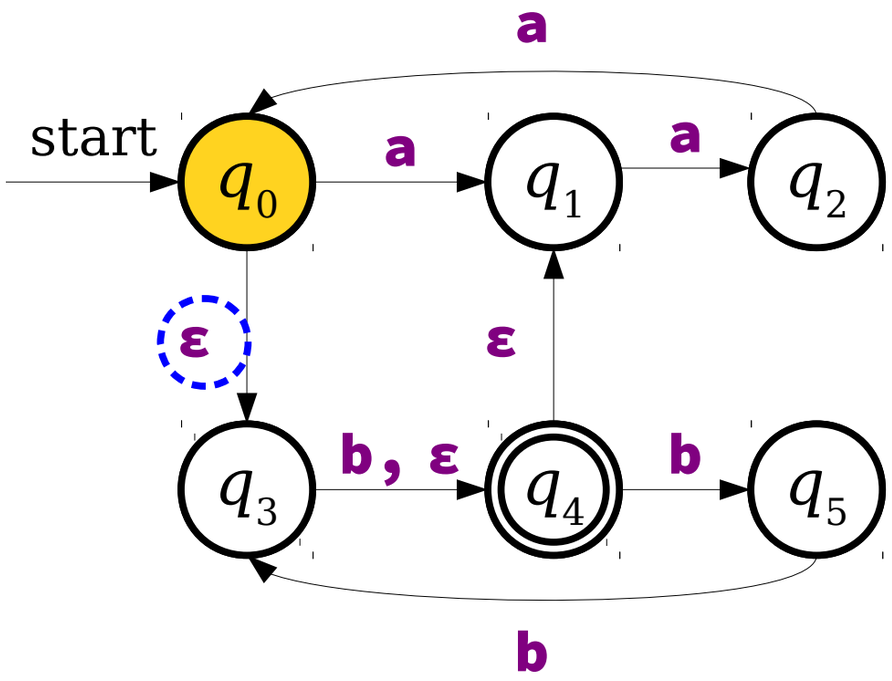

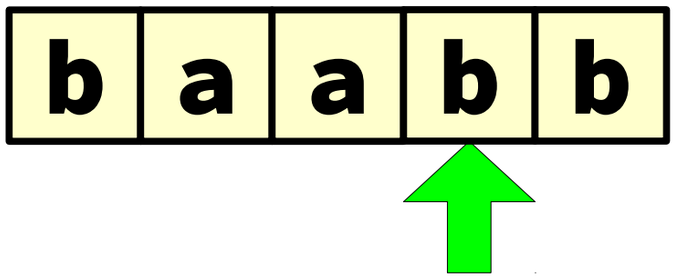

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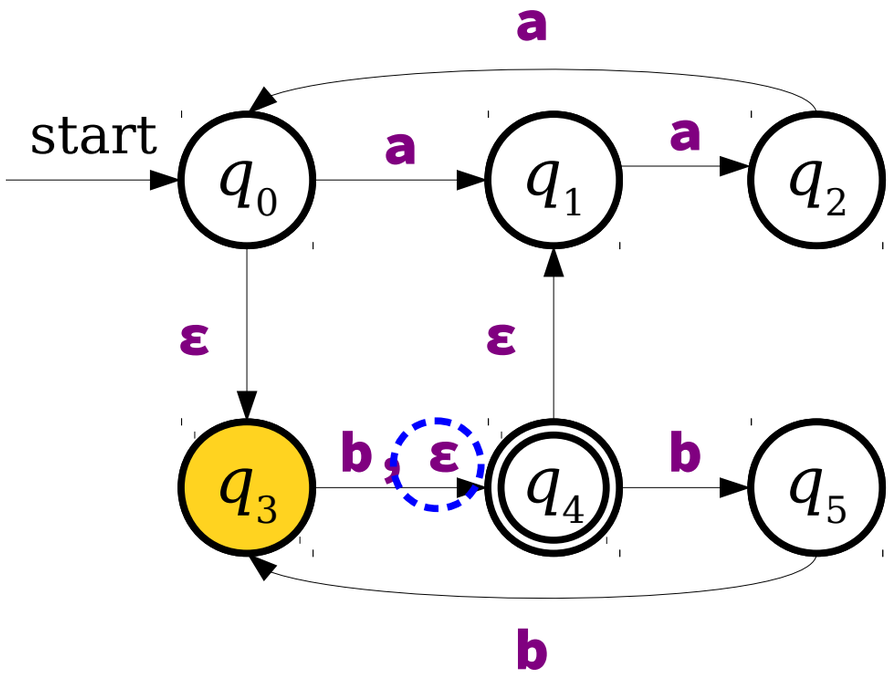

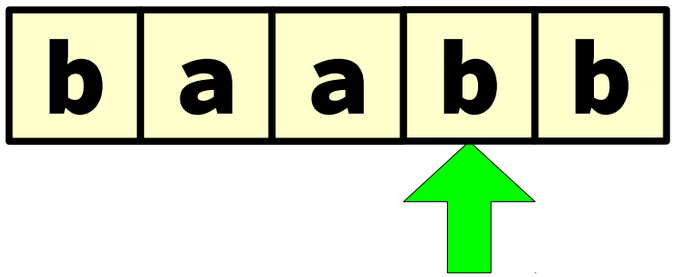

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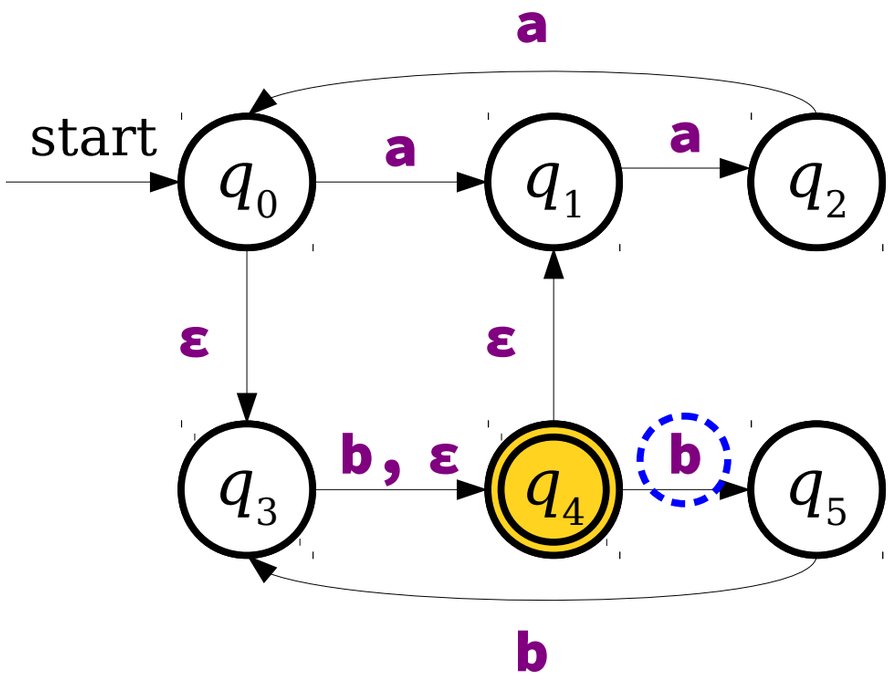

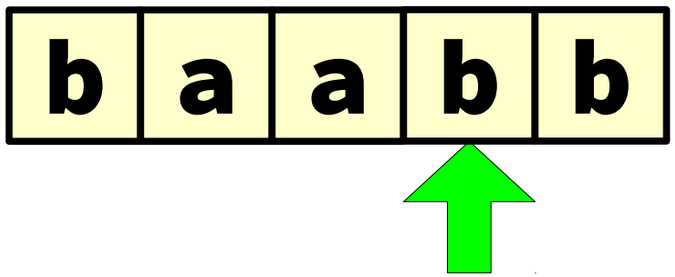

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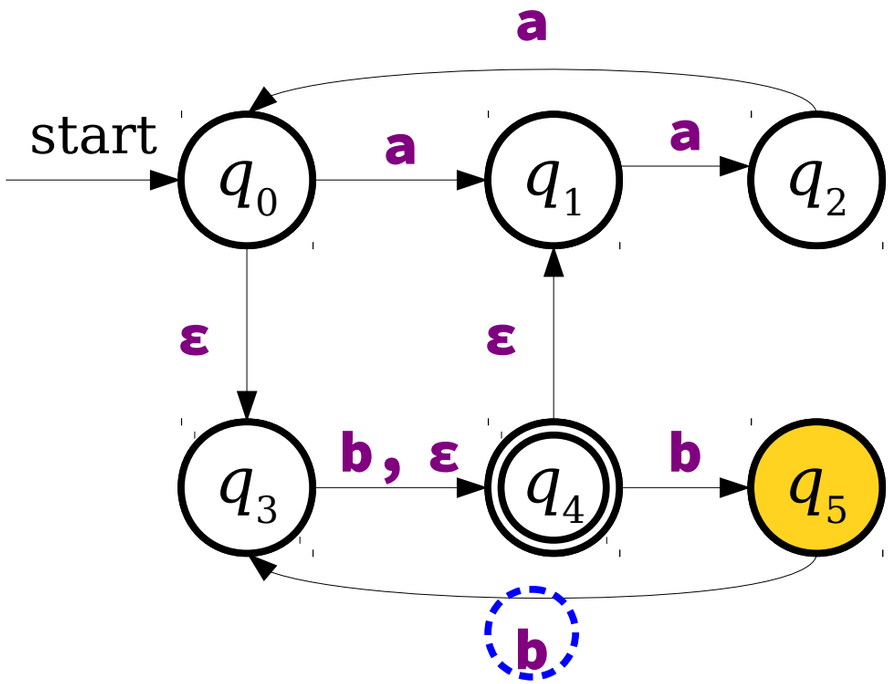

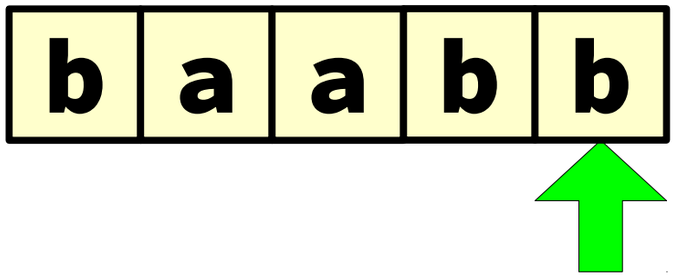

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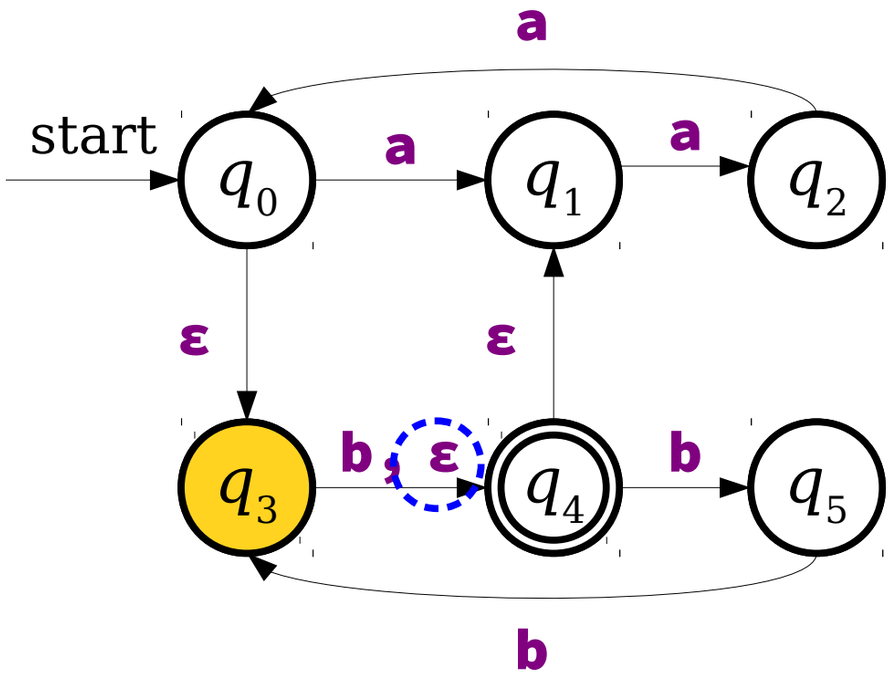

b	a	a	b	b

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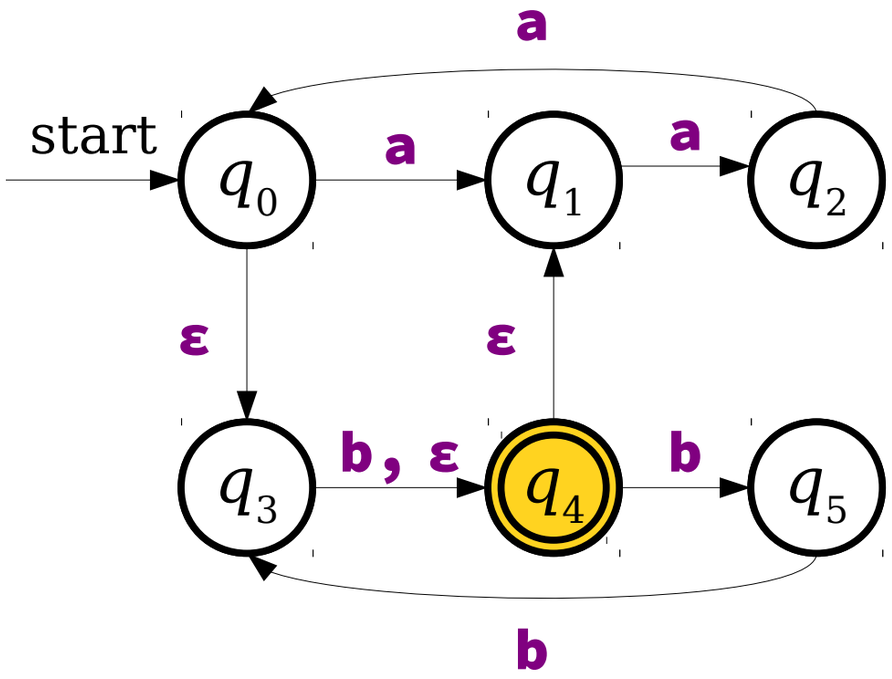

b	a	a	b	b

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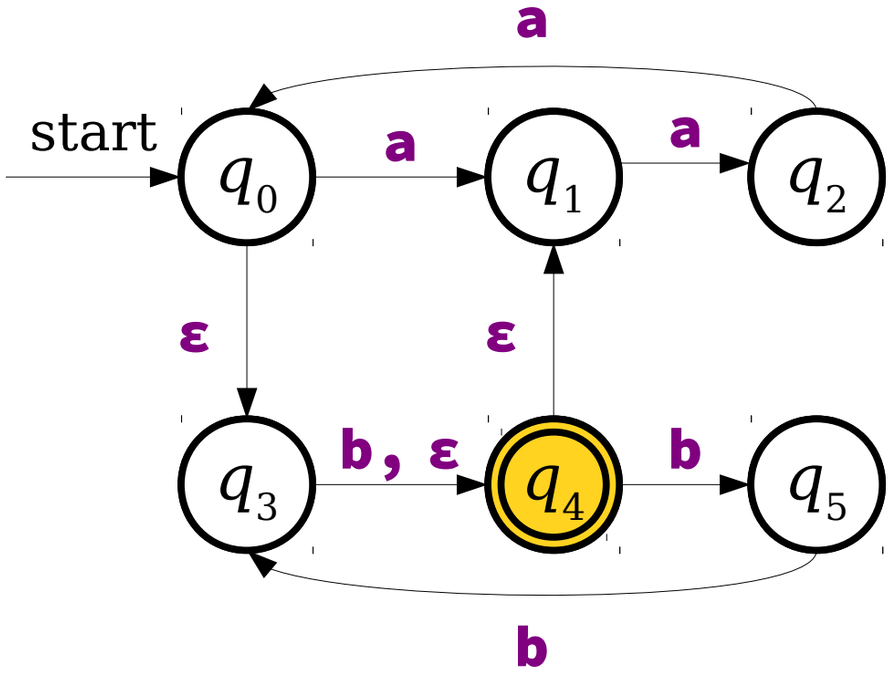

b	a	a	b	b

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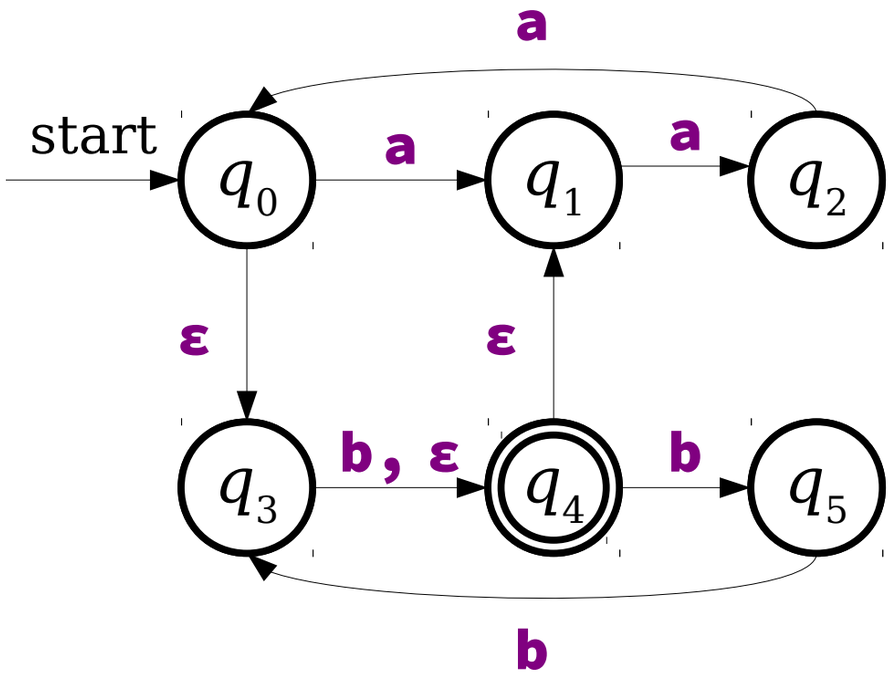


Not at all fun or rewarding exercise: what is the language of this NFA?

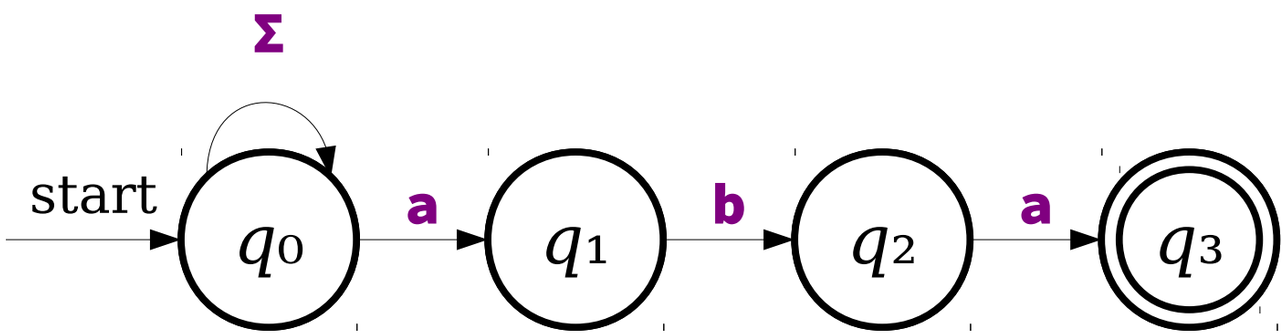
ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
- NFAs are not *required* to follow ε-transitions. It's simply another option at the machine's disposal.

Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers. How can we build up an intuition for them?
- There are two particularly useful frameworks for interpreting nondeterminism:
  - ***Perfect positive guessing***
  - ***Massive parallelism***

Perfect Positive Guessing



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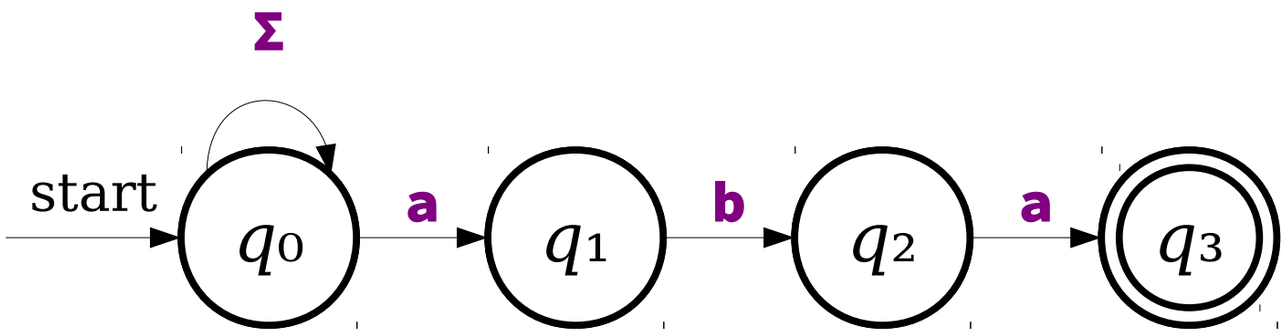

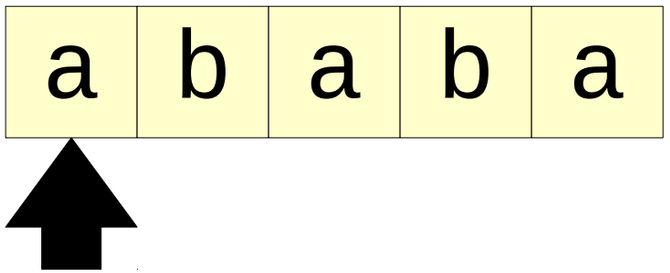

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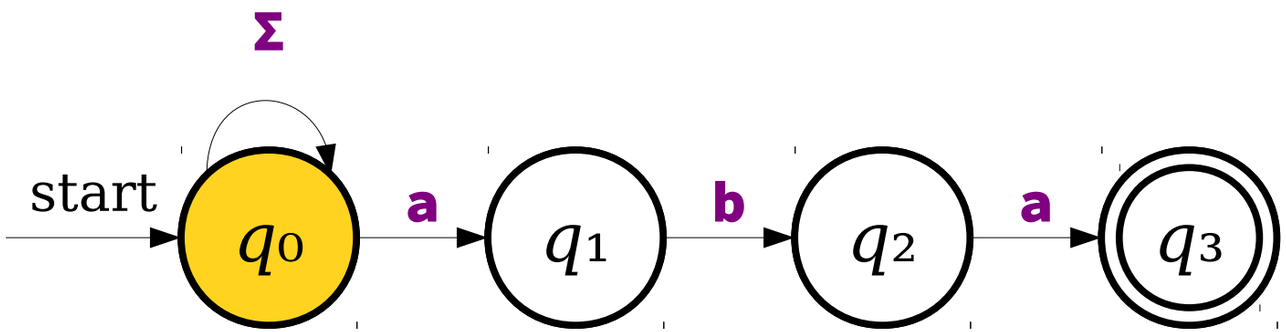

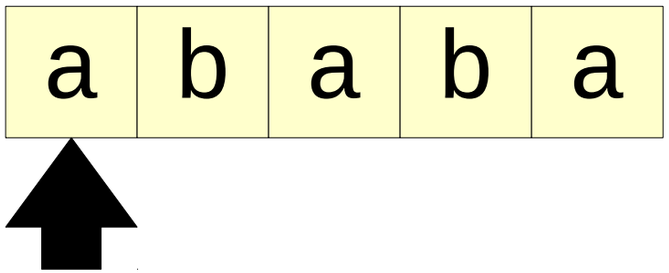

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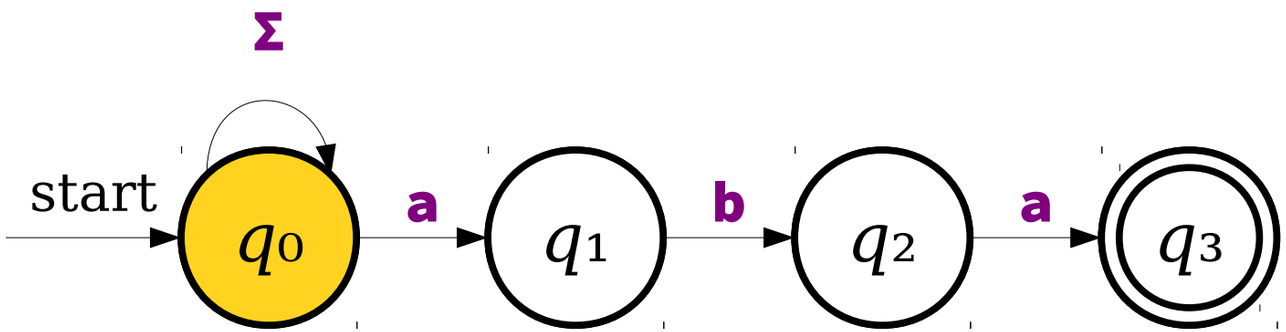

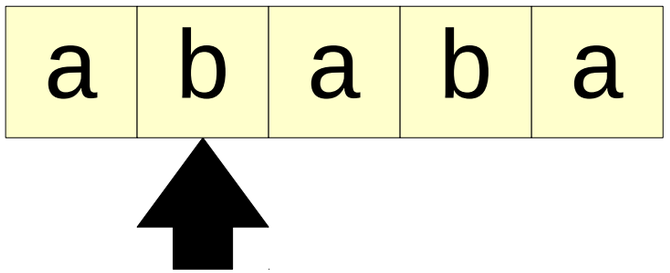

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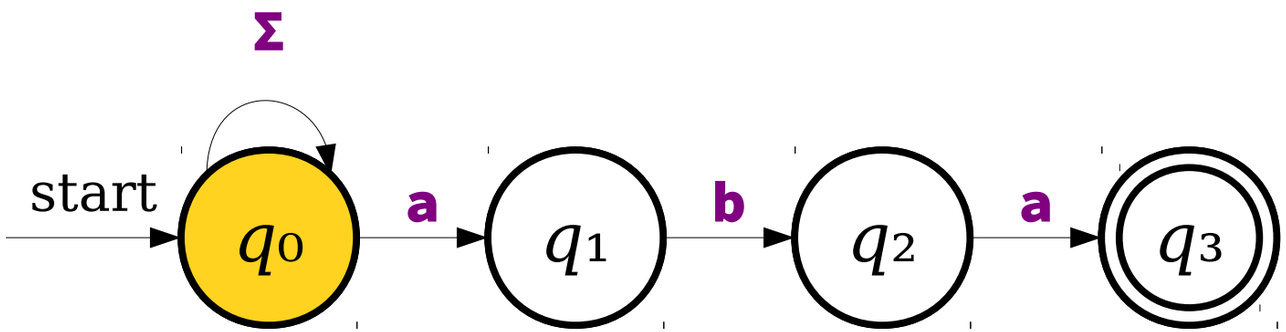

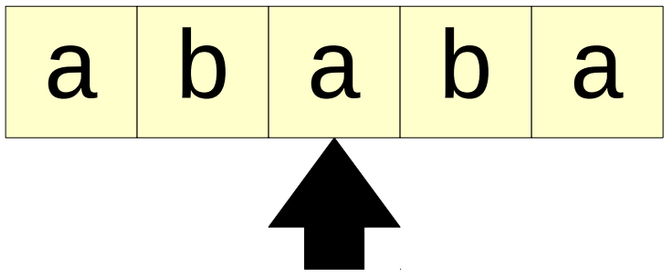

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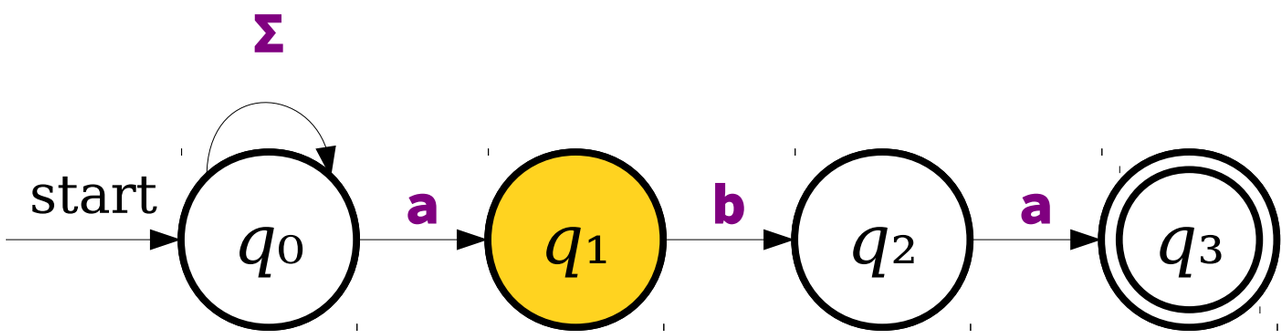

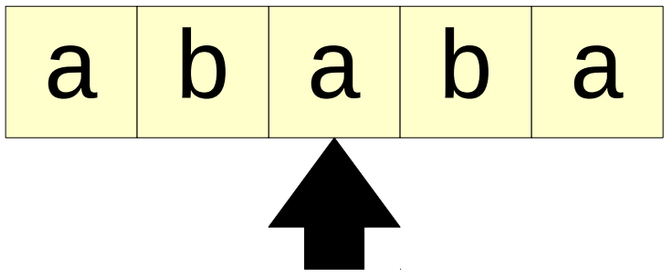

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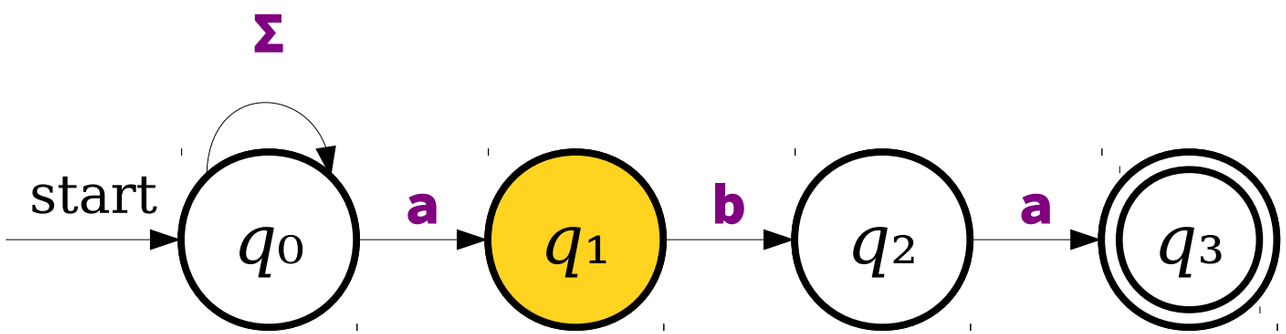

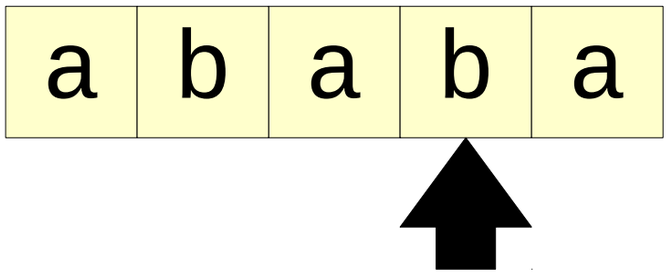

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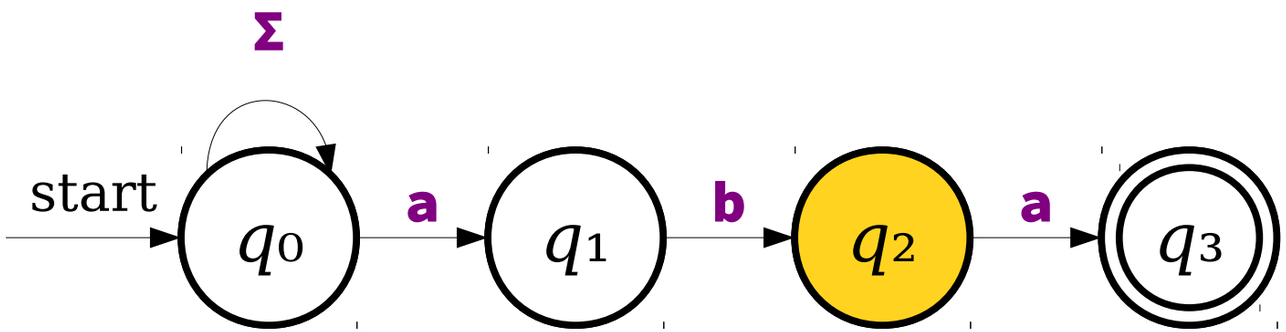

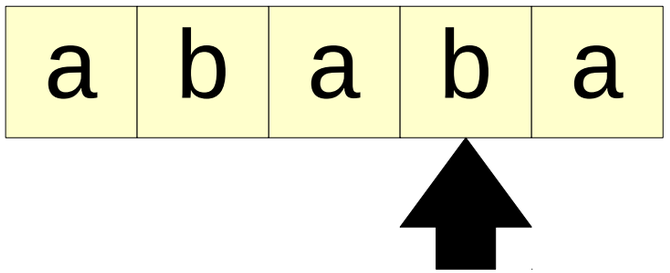

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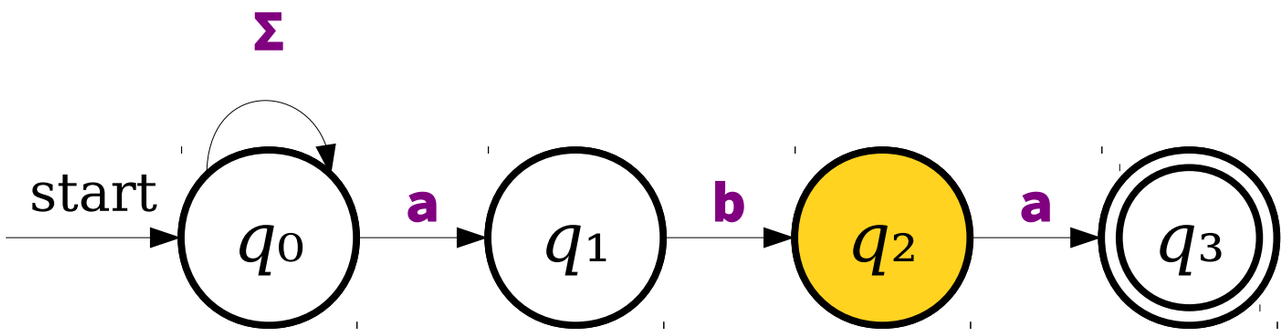

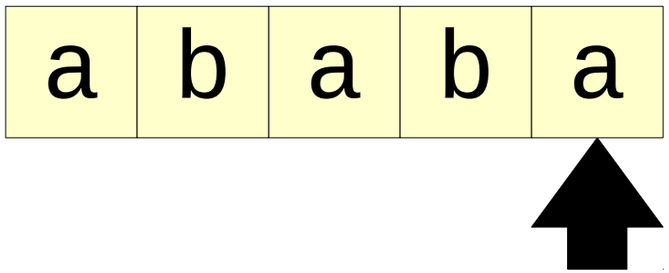

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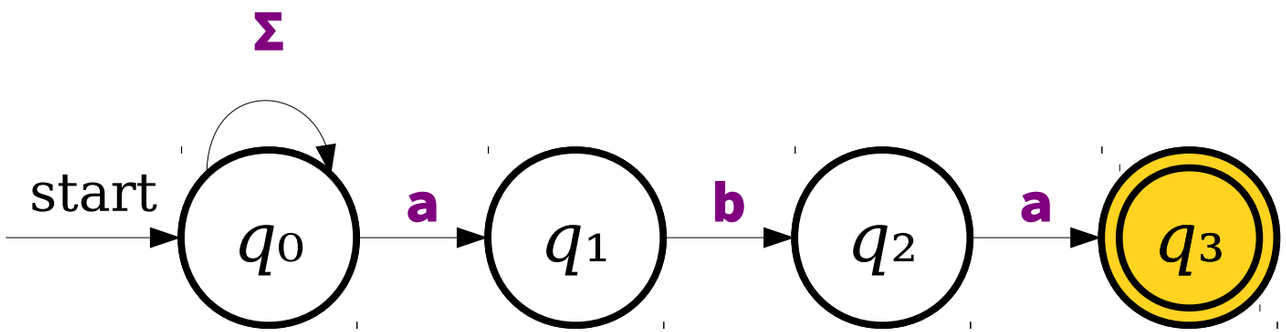

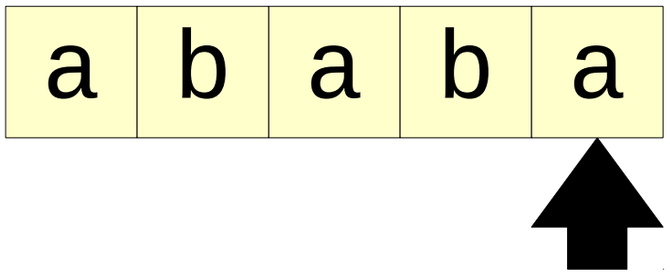

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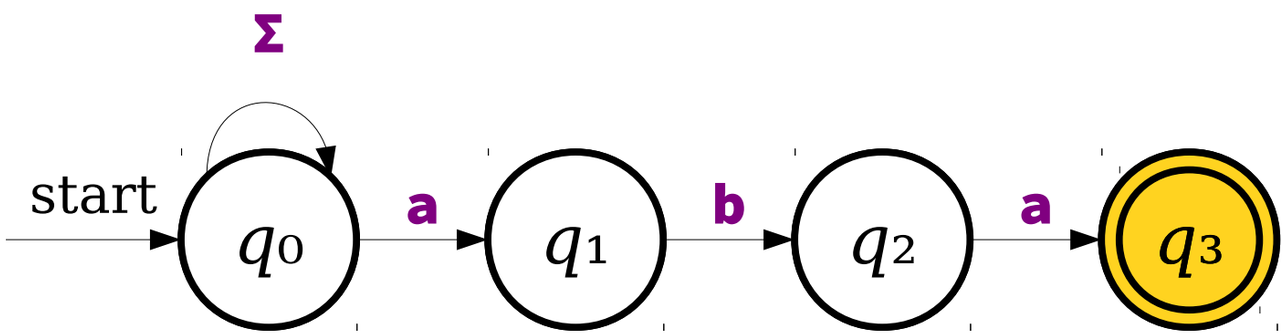
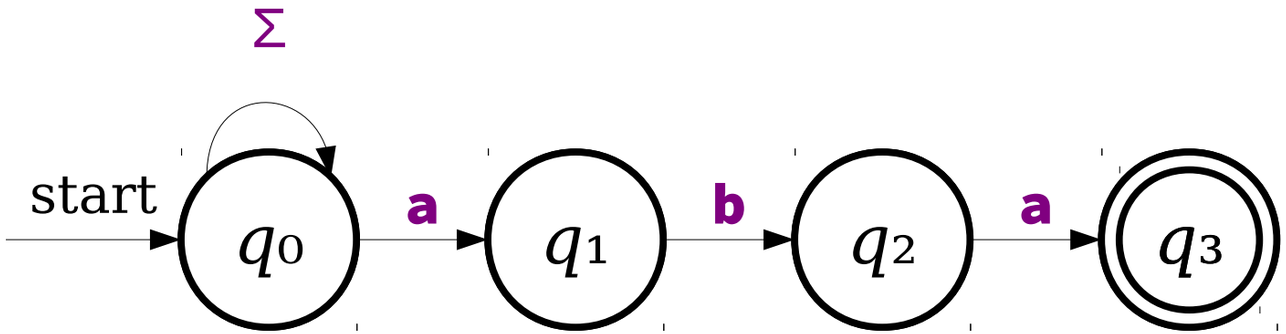


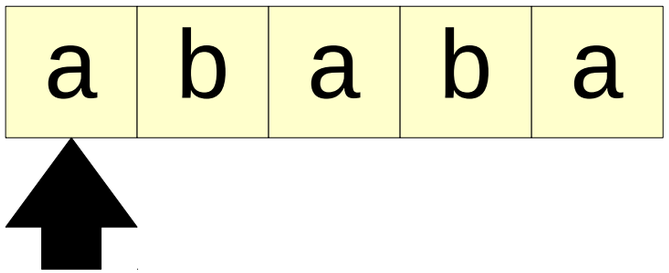
a	b	a	b	a

Perfect Positive Guessing

- We can view nondeterministic machines as having ***Magic Superpowers*** that enable them to guess choices that lead to an accepting state.
  - If there is at least one choice that leads to an accepting state, the machine will guess it.
  - If there are no choices, the machine guesses any one of the wrong guesses.
- There is no known way to physically model this intuition of nondeterminism – this is quite a departure from reality!

Massive Parallelism





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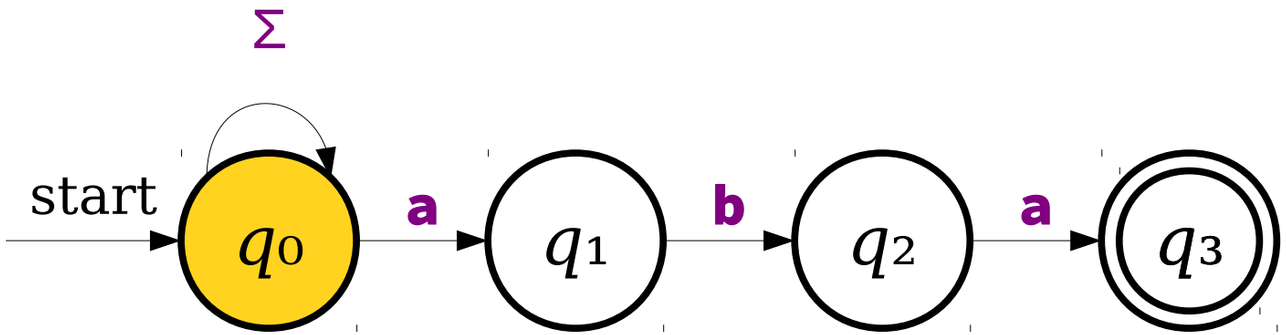

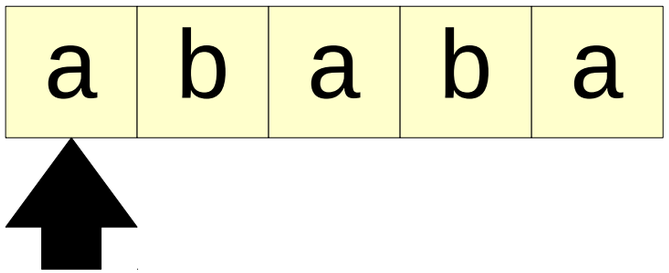

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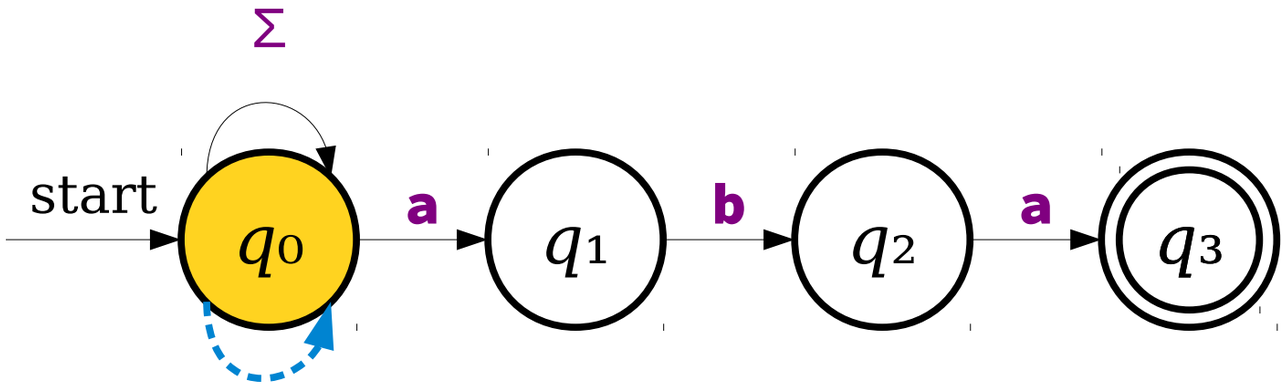

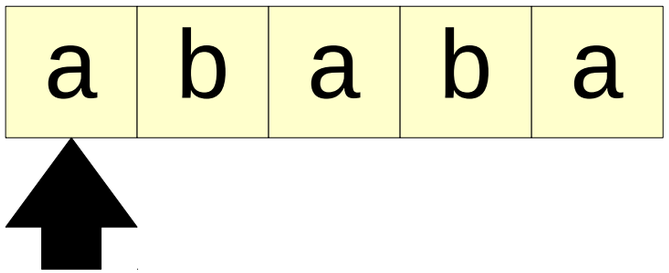

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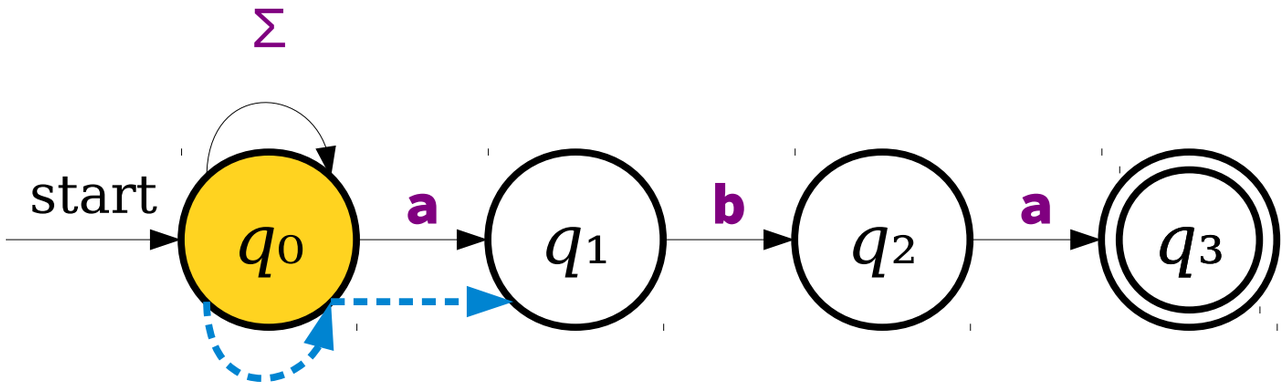

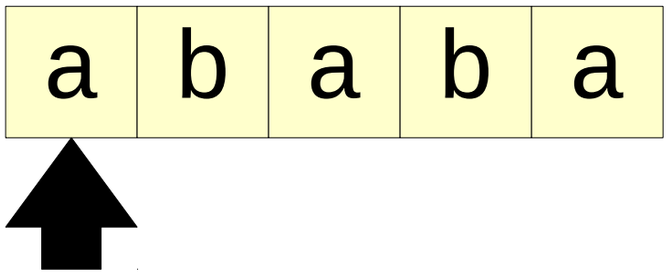

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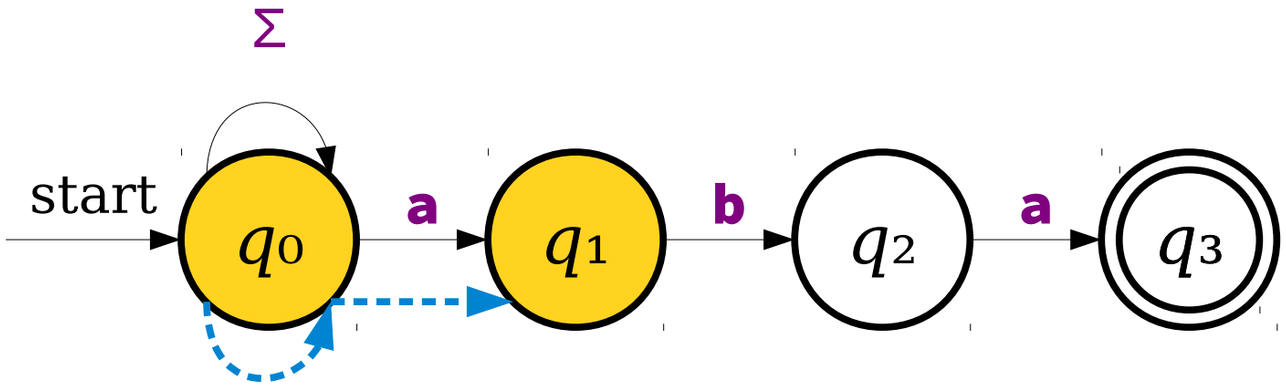

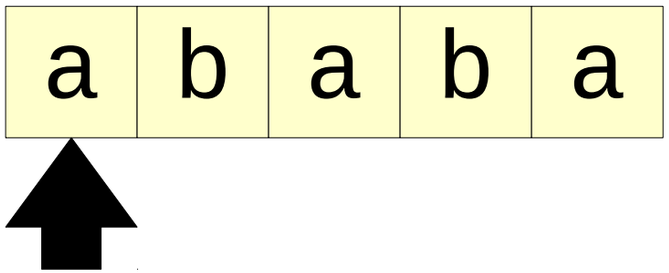

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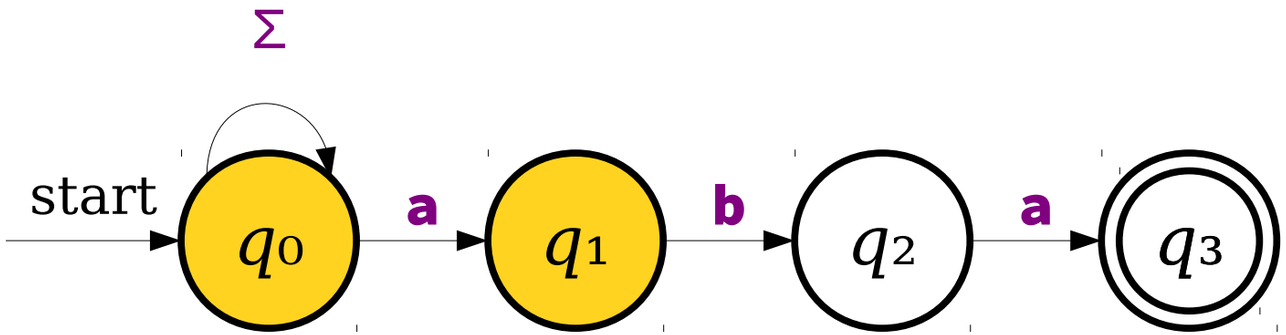

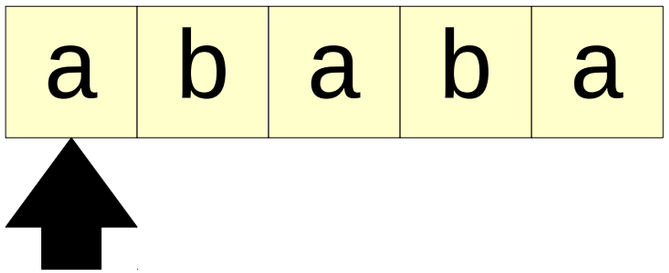

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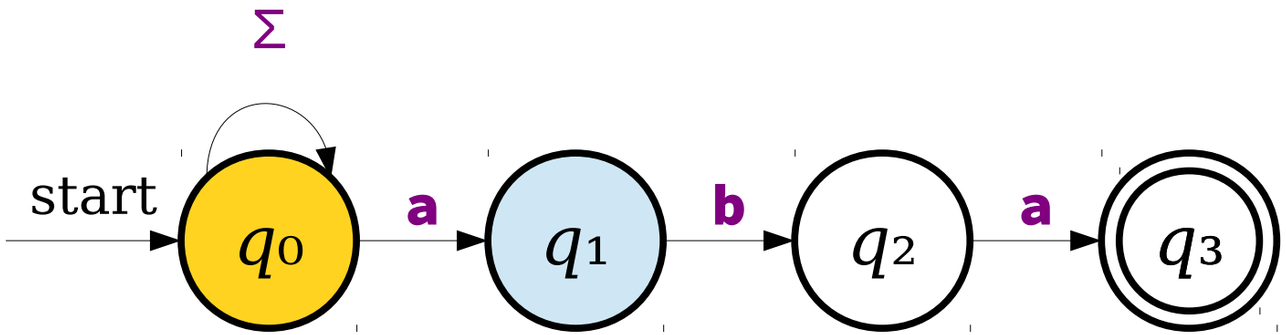

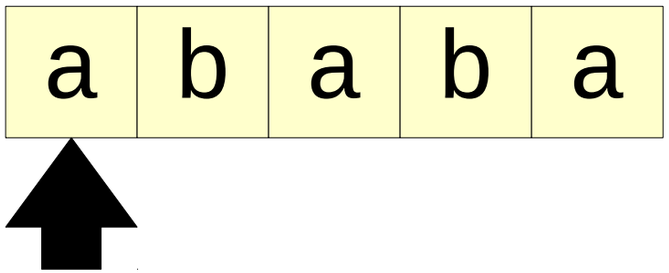

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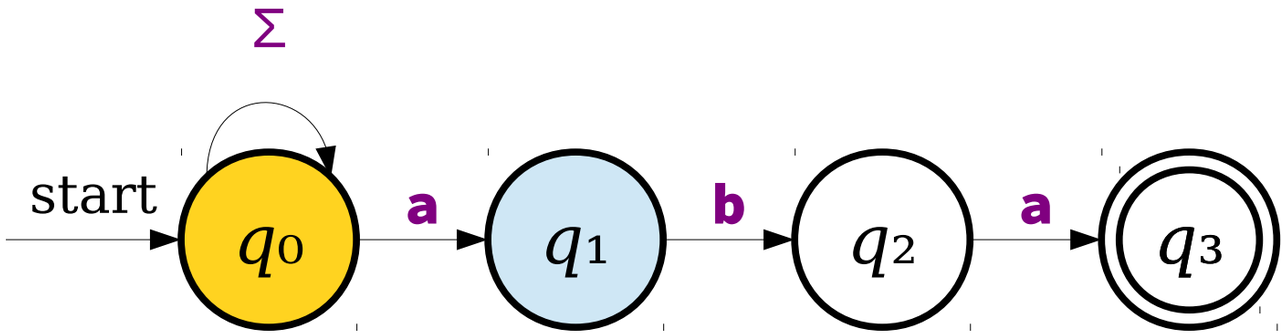

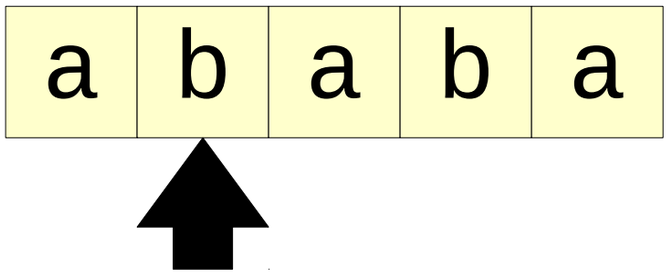

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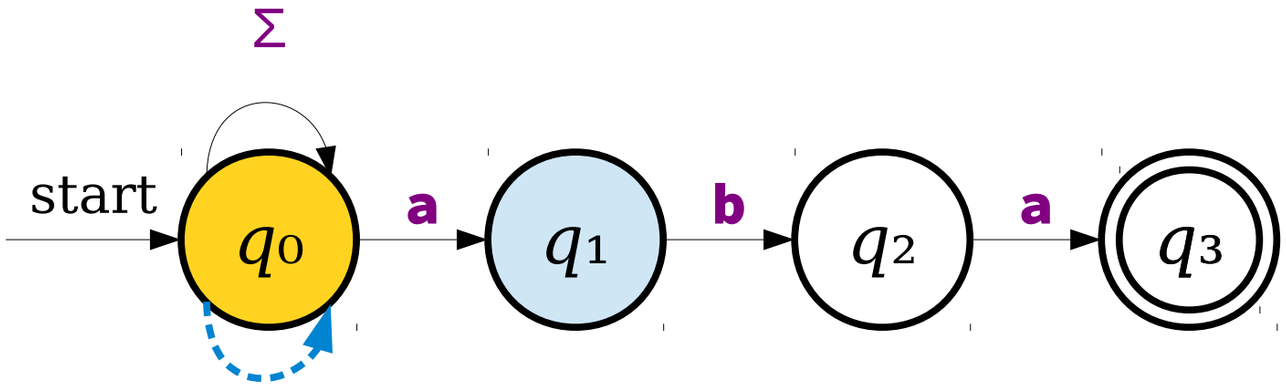

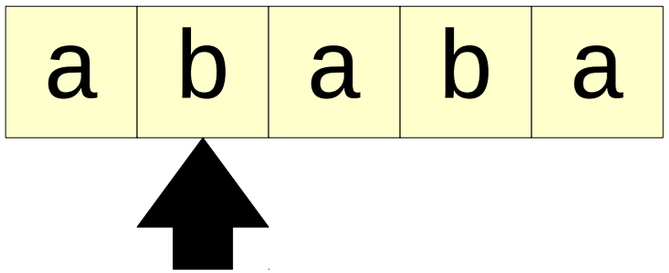

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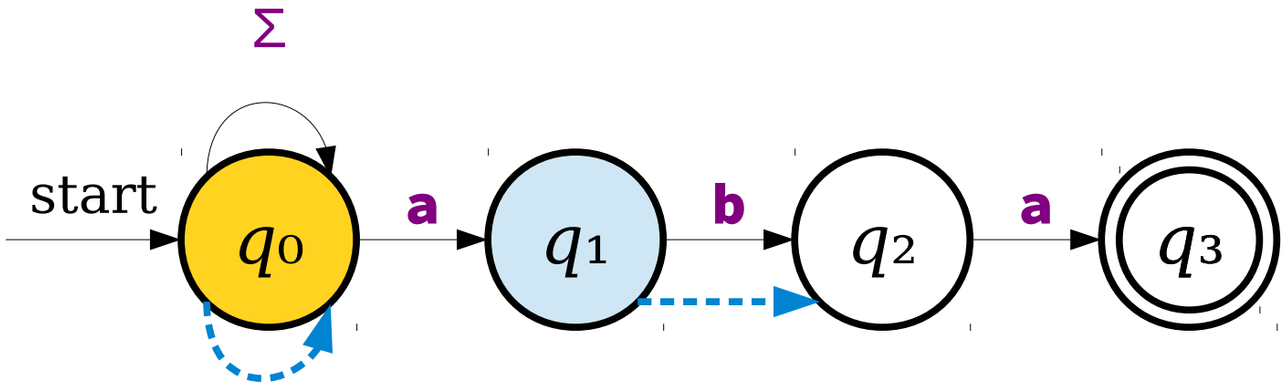

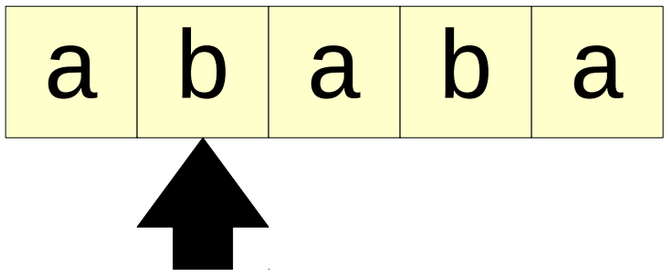

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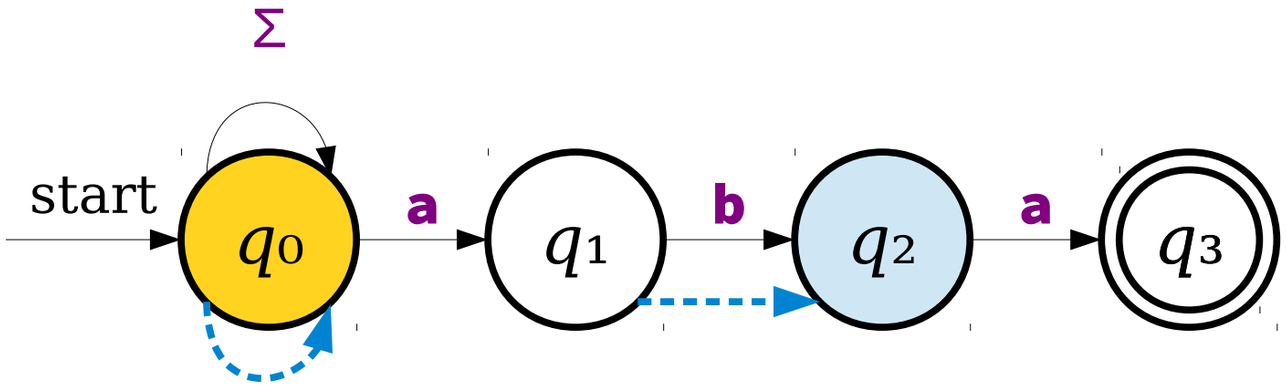

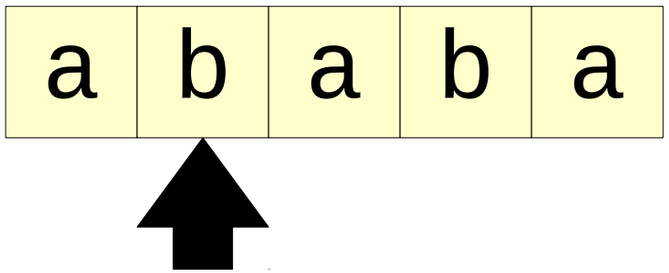

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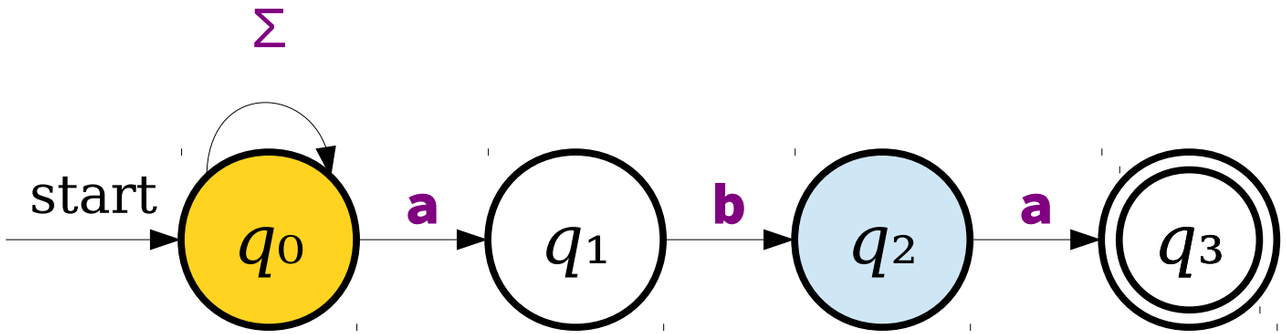

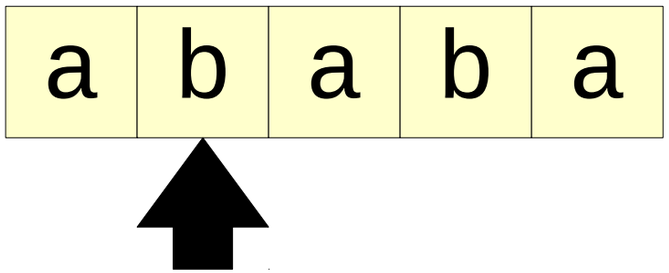

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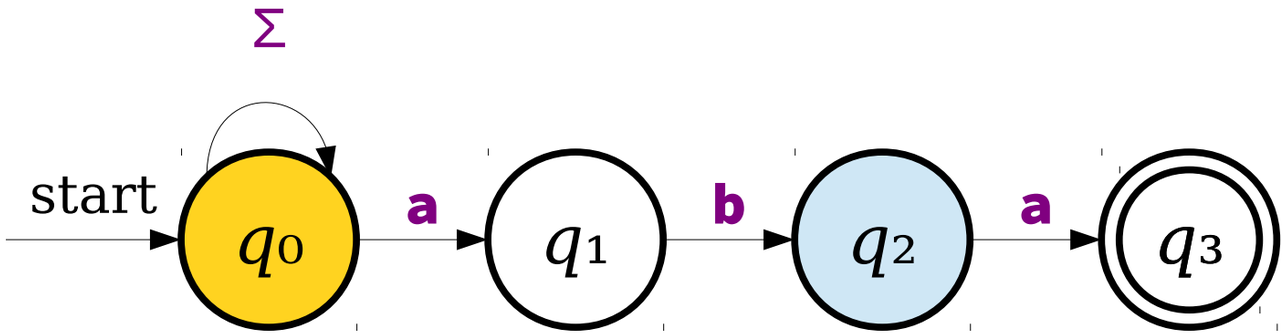

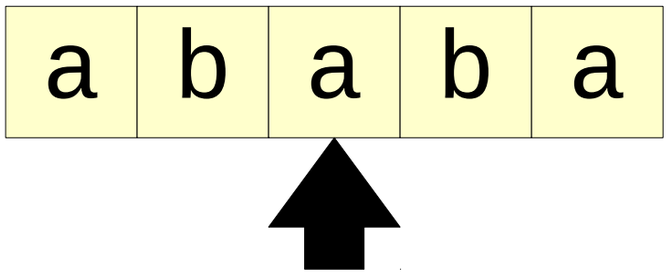

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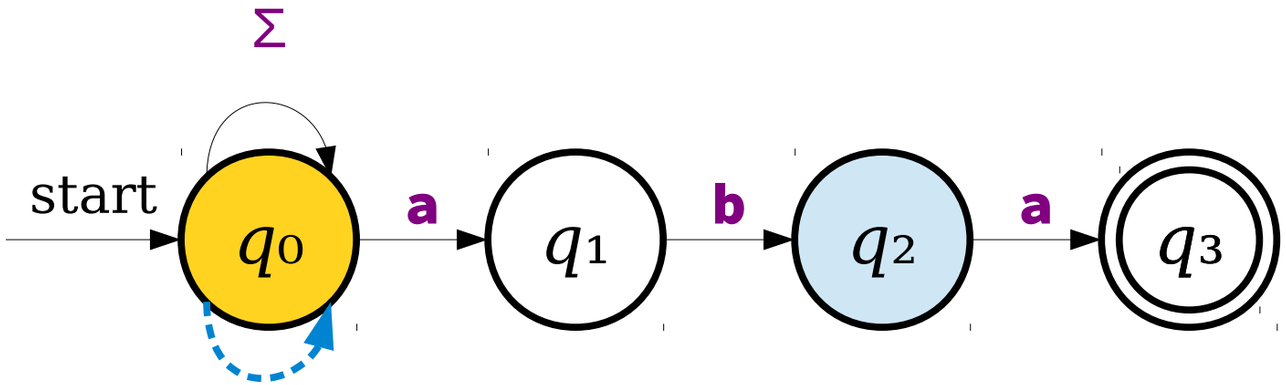

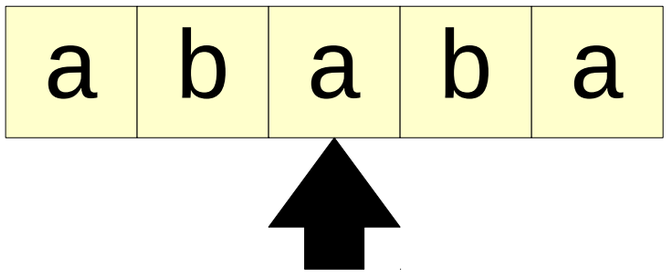

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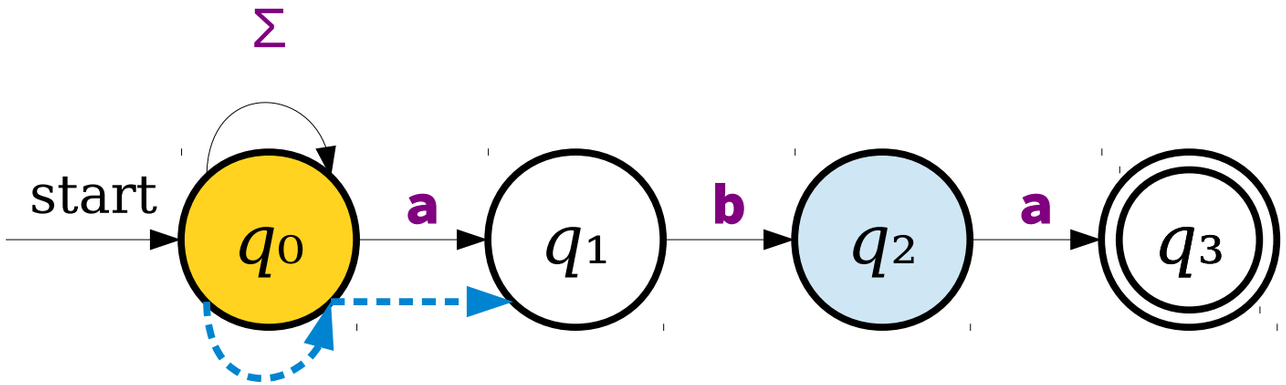

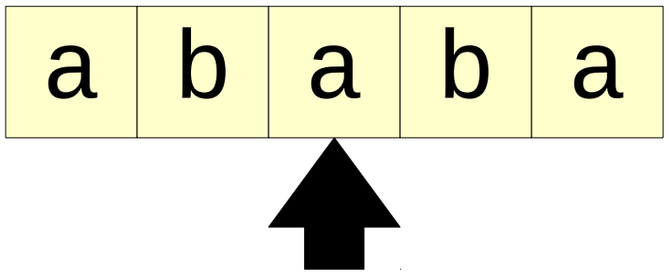

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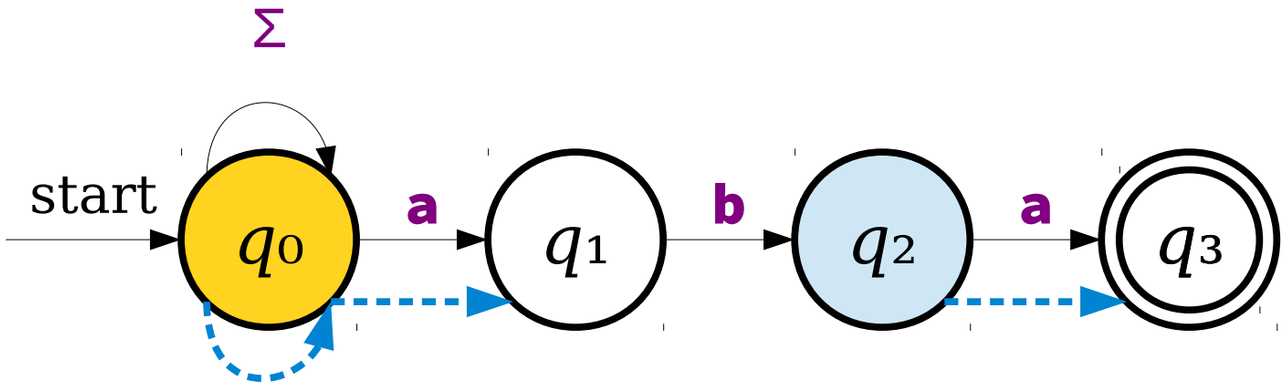

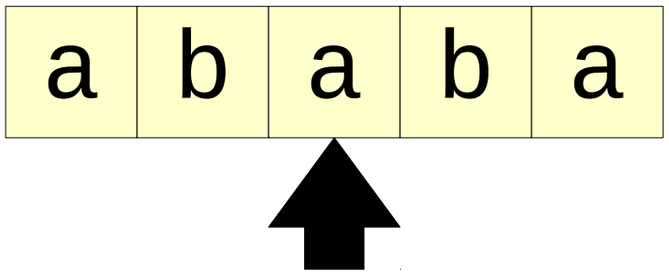

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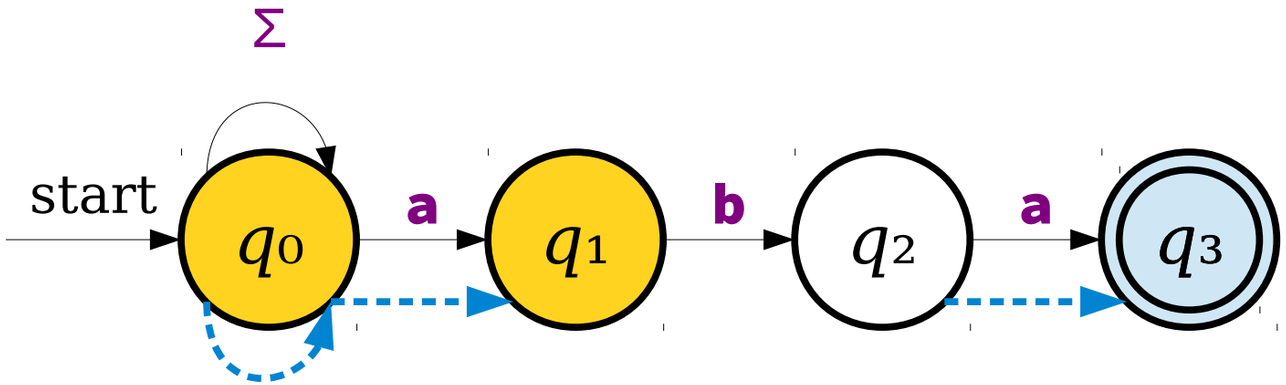

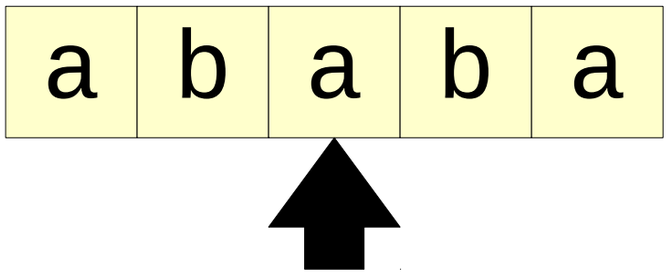

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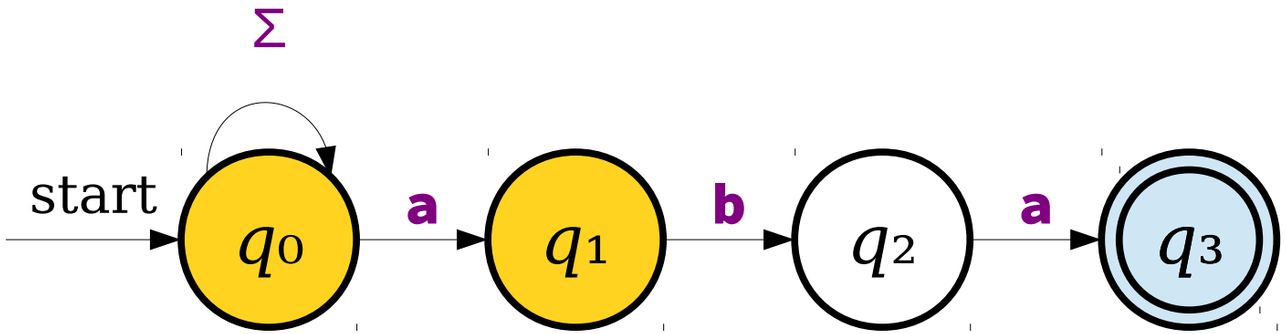

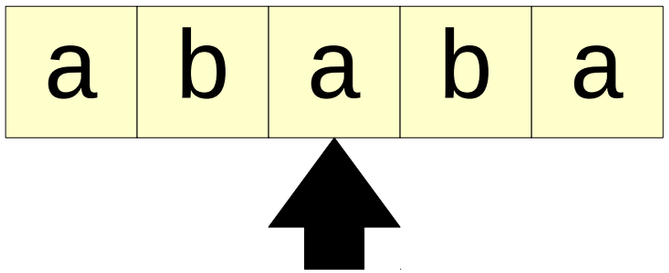

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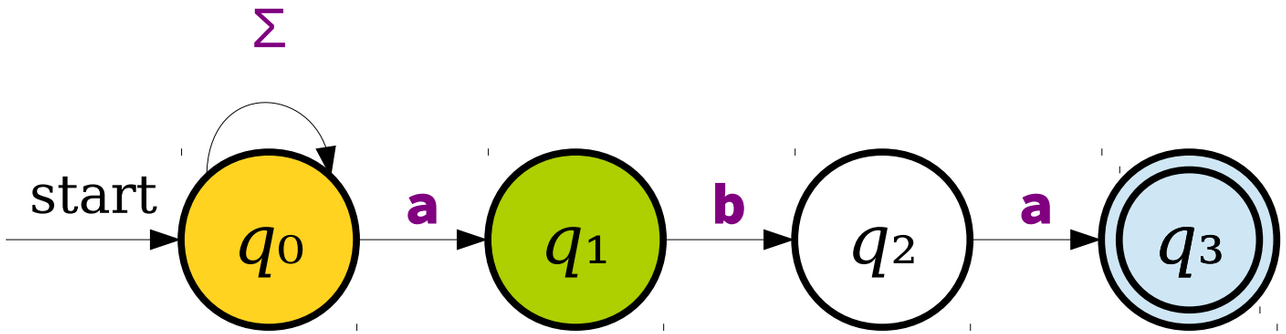

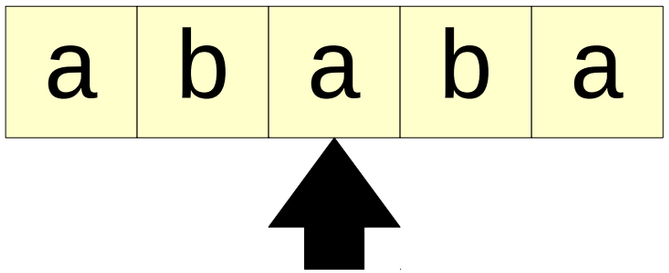

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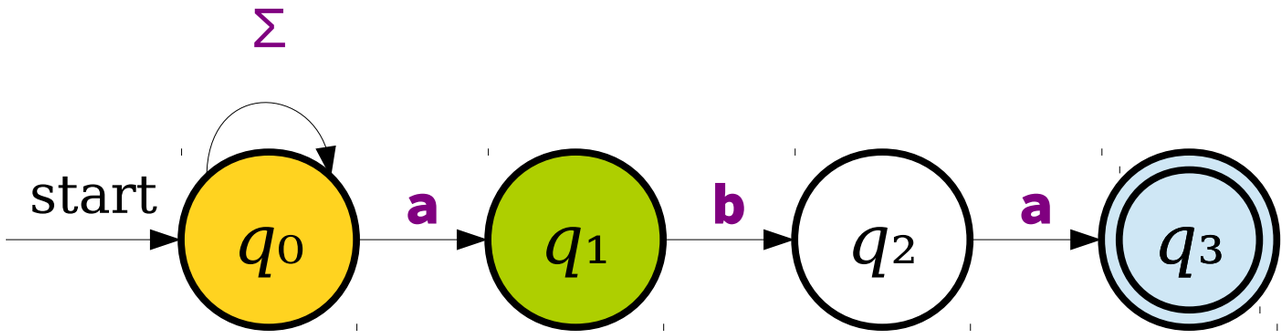

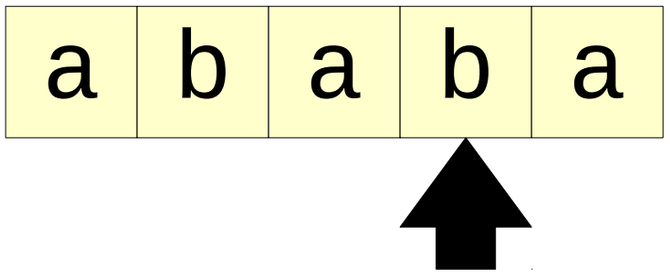

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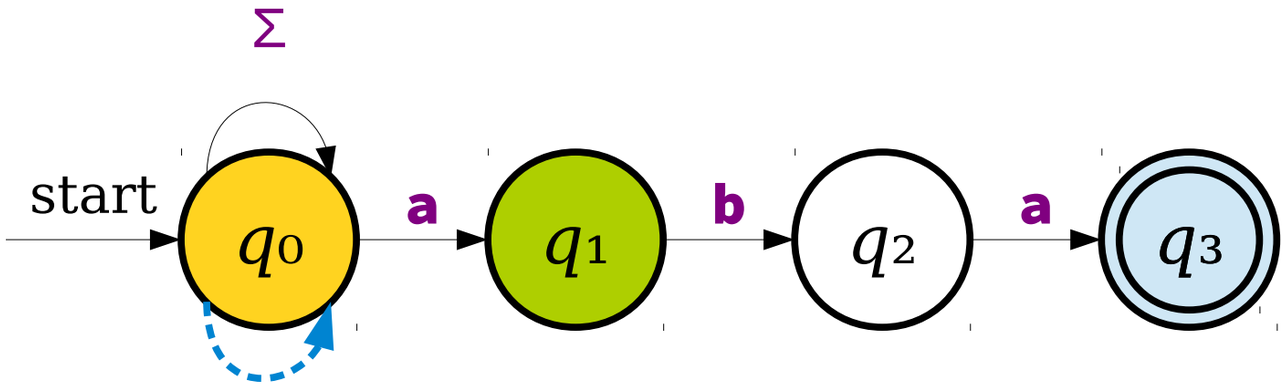

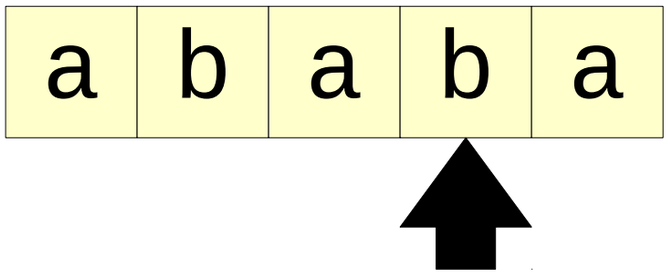

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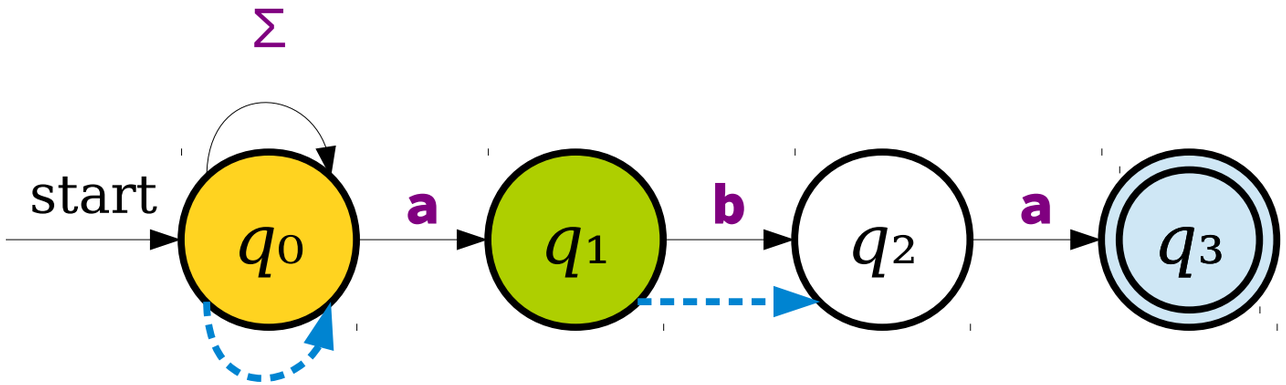

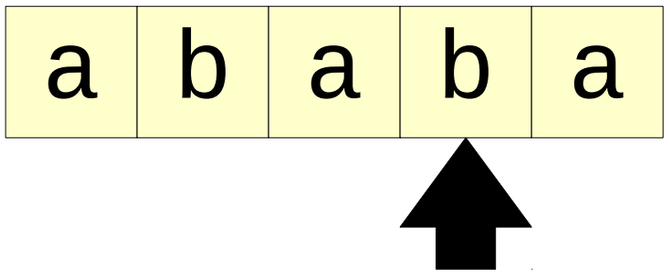

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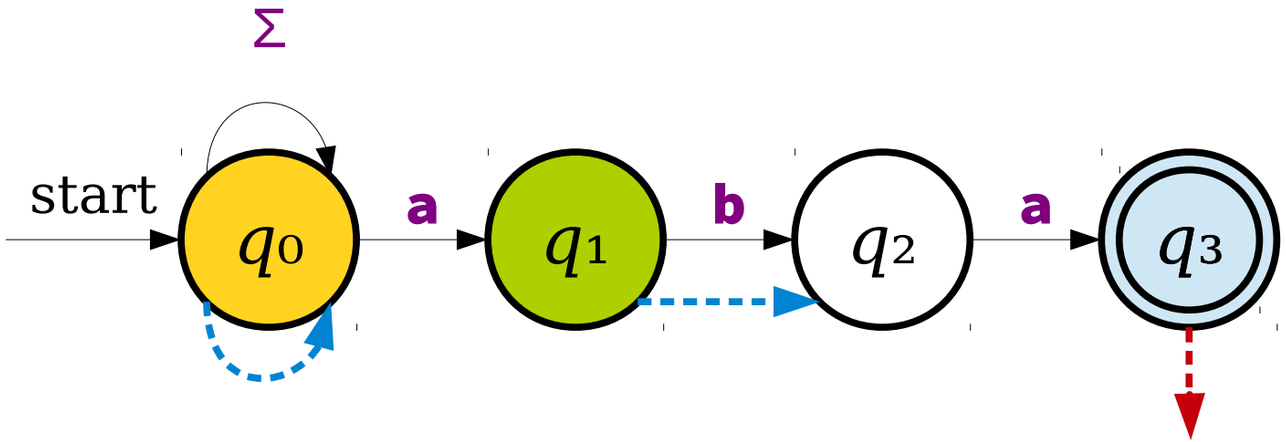

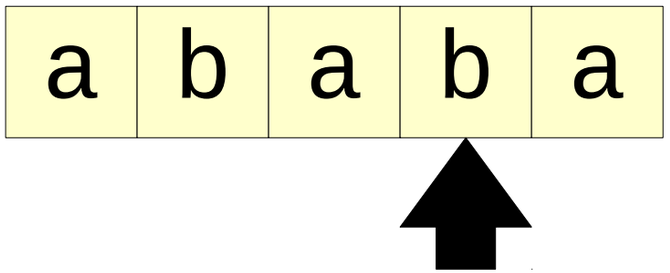

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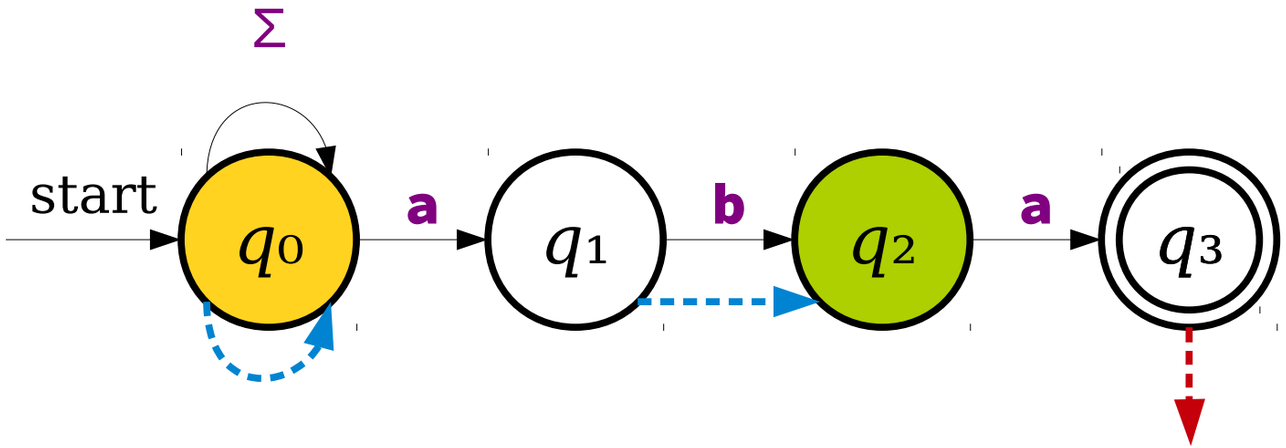

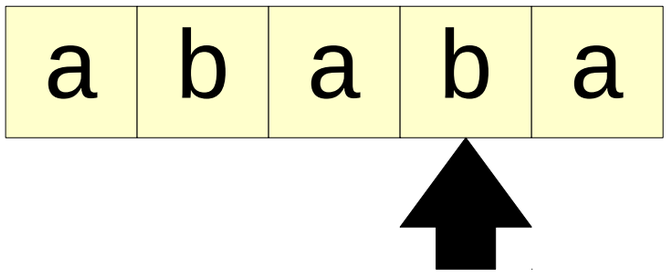

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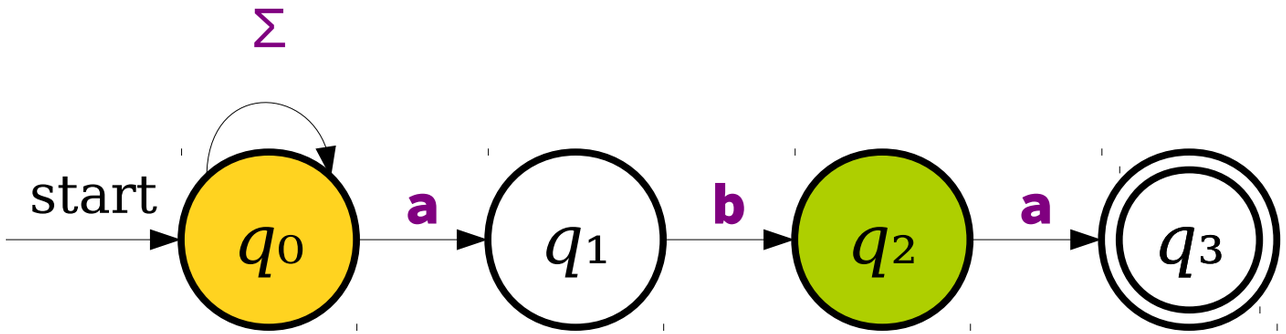

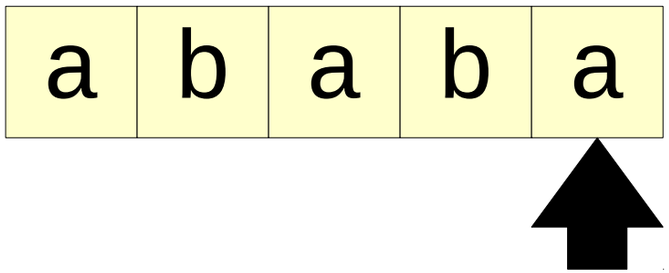

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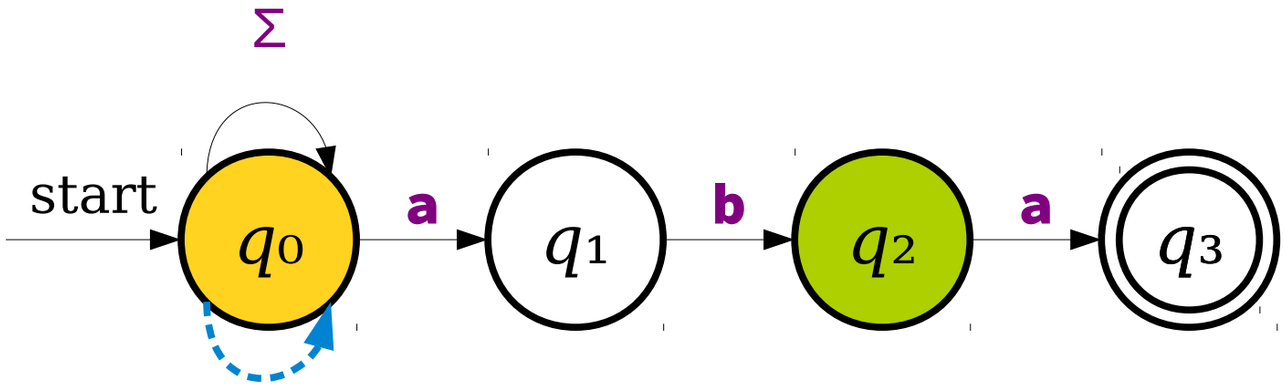

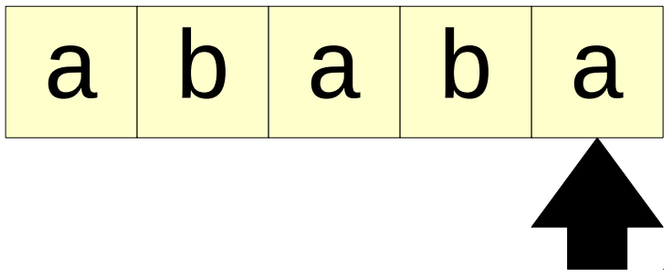

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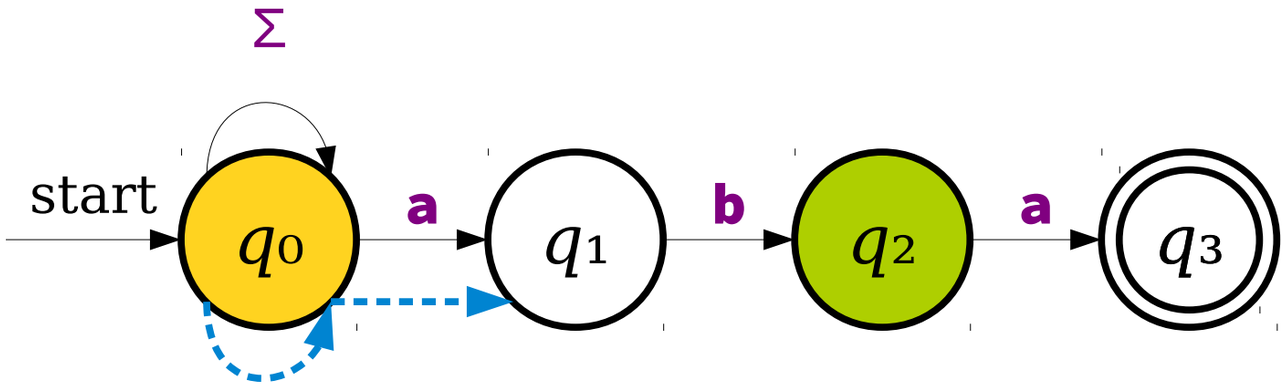

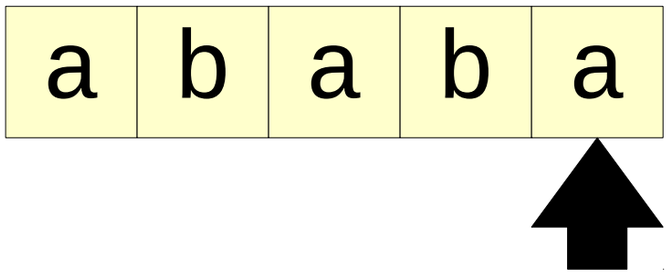

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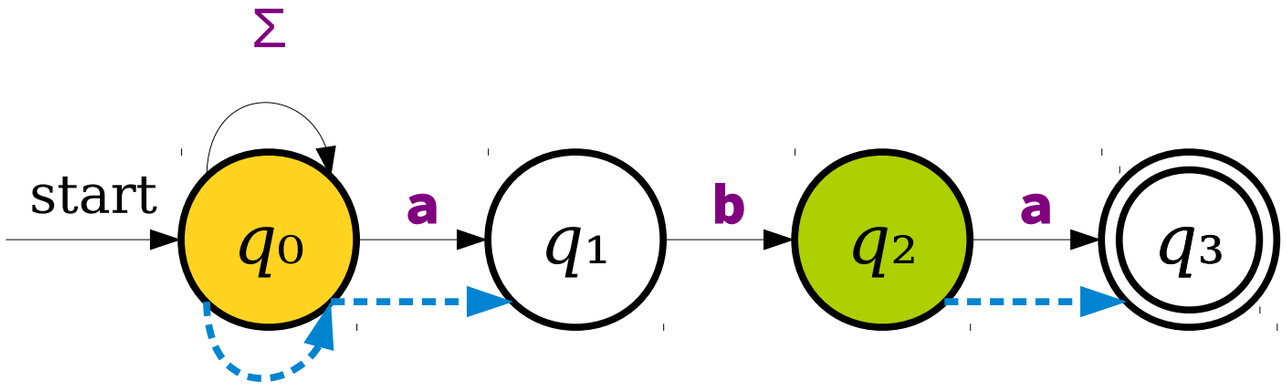

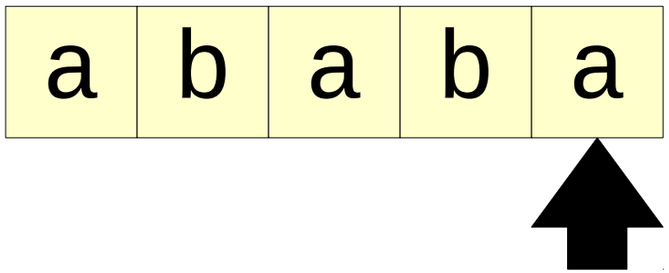

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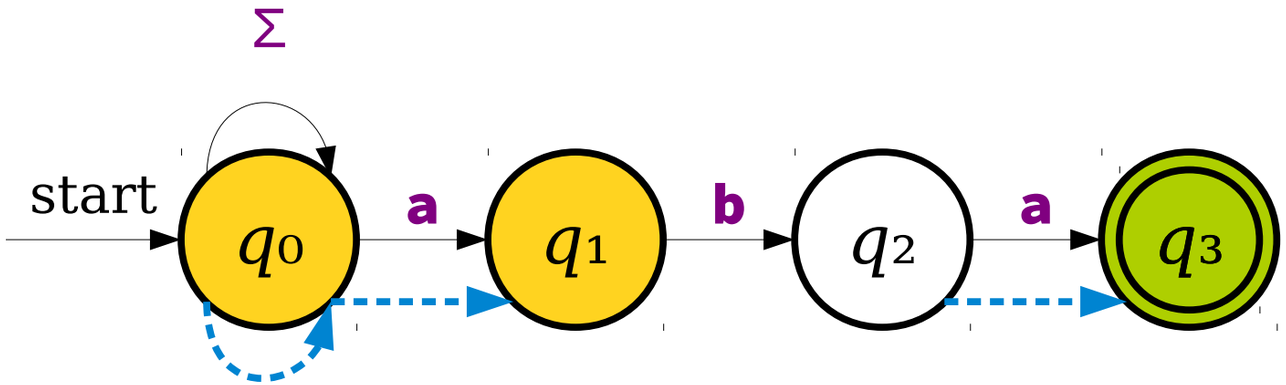

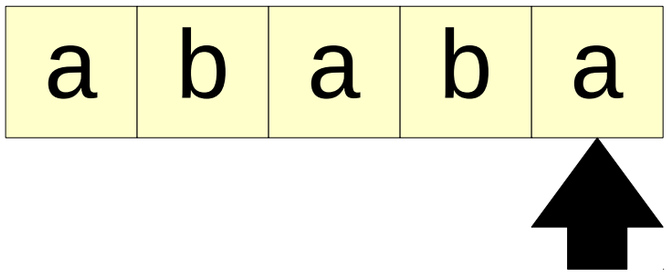

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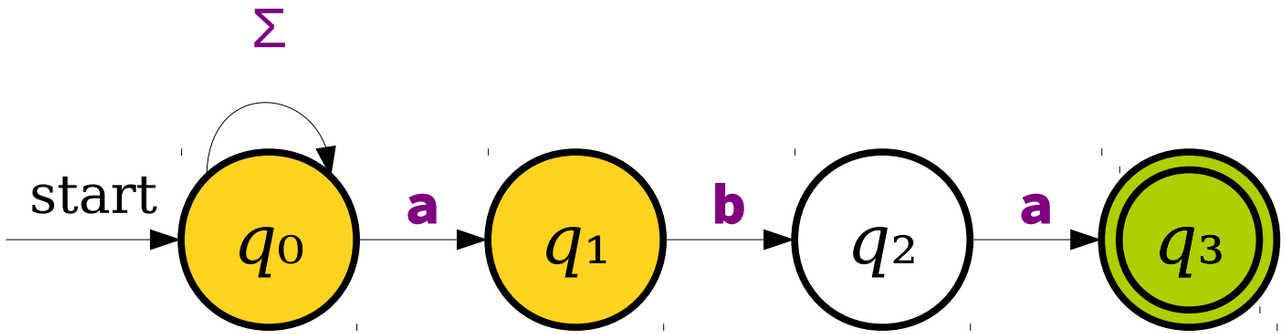

a	b	a	b	a

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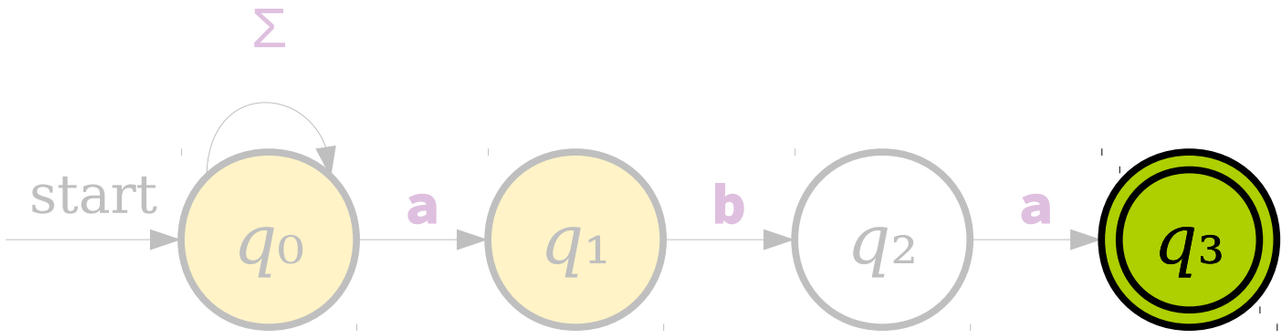
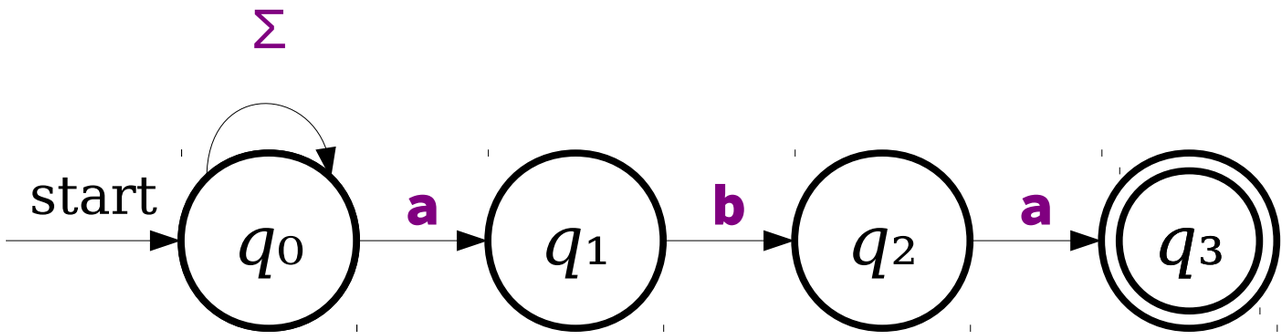
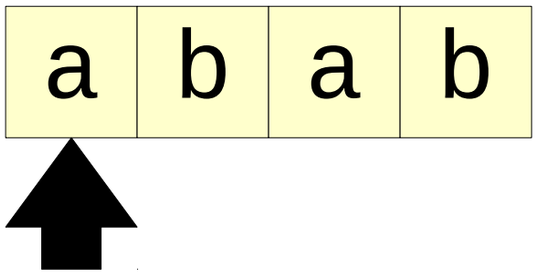


We're in at least one accepting state, so there's some path that gets us to an accepting state.

a	b	a	b	a

Massive Parallelism





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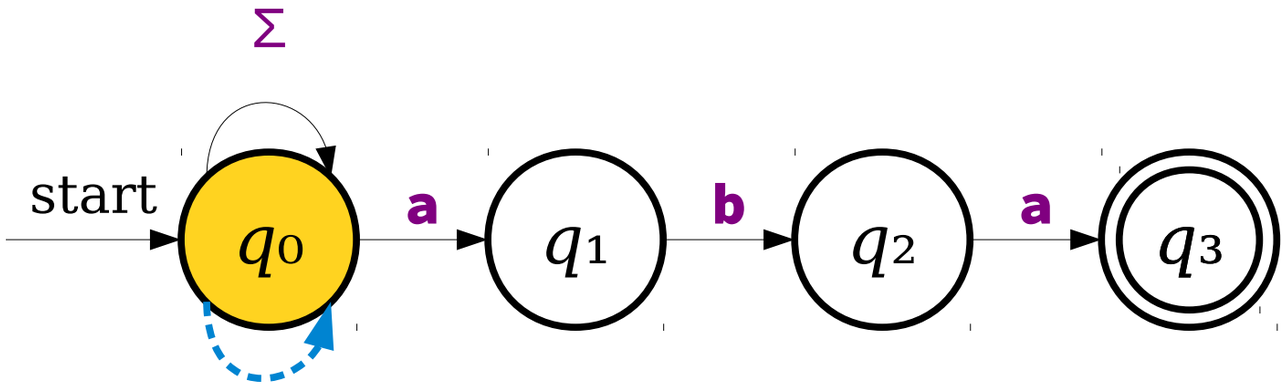

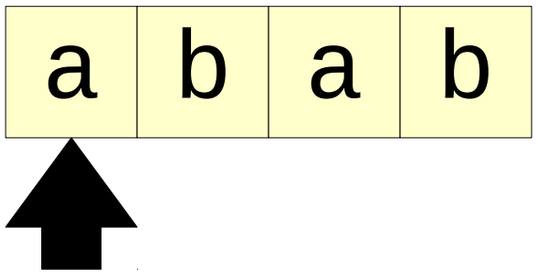

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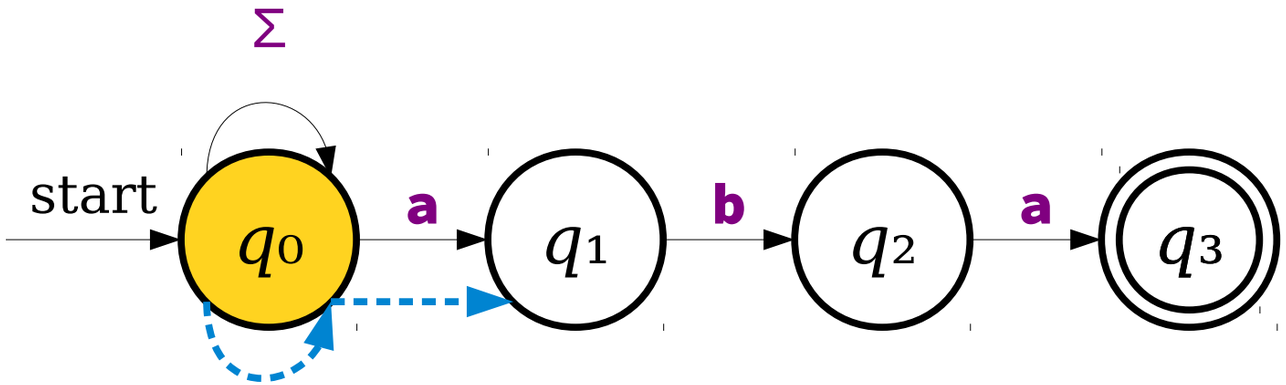

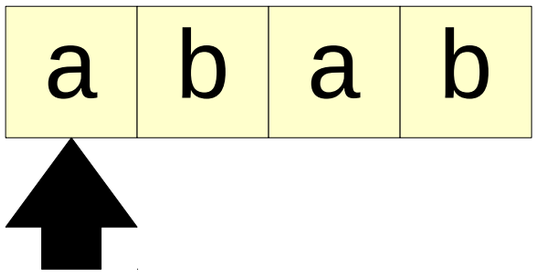

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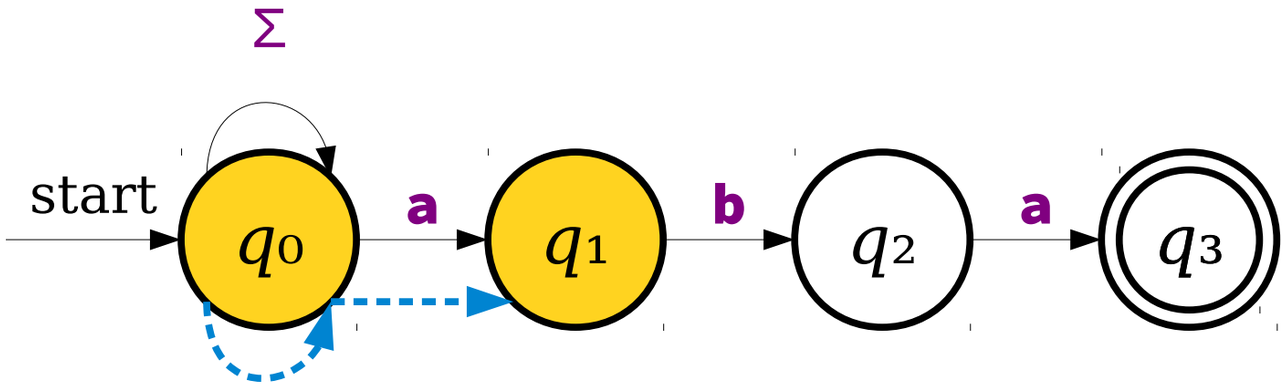

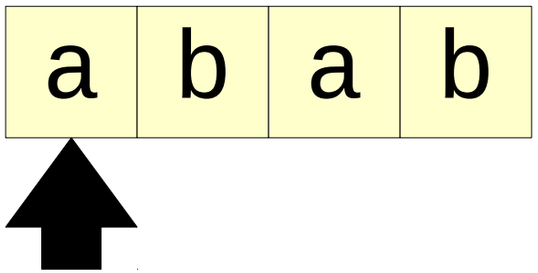

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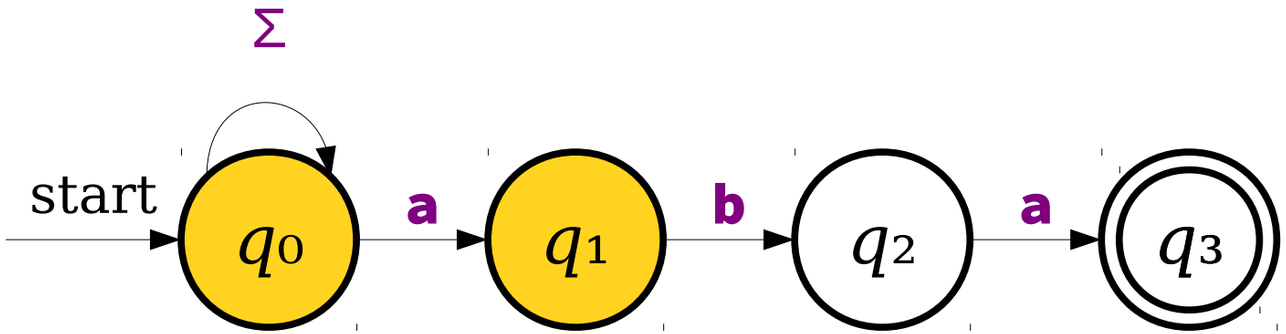

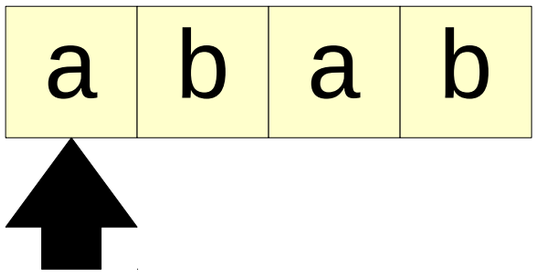

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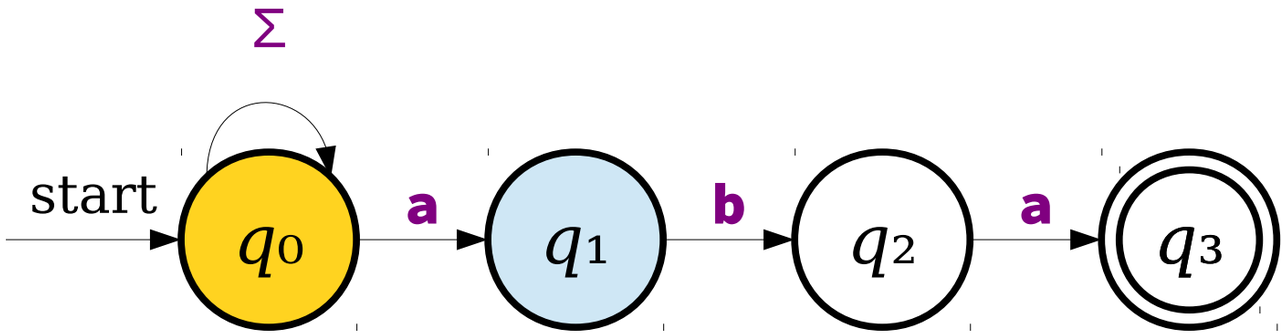

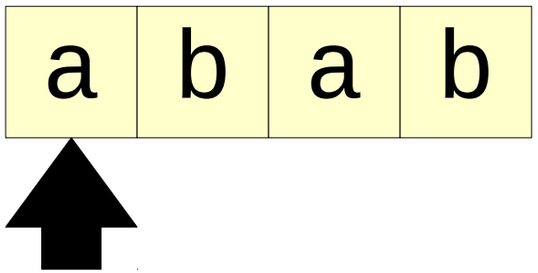

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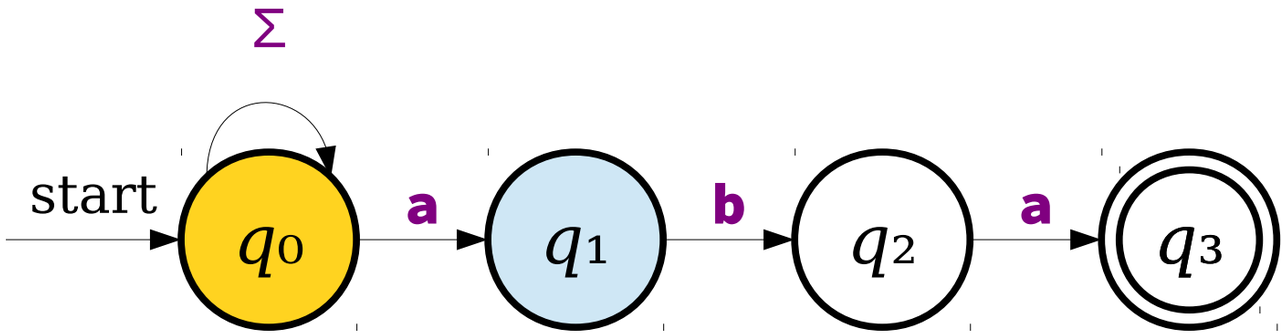

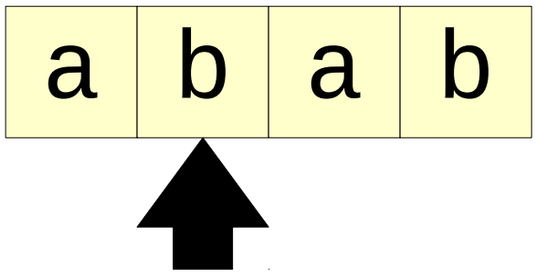

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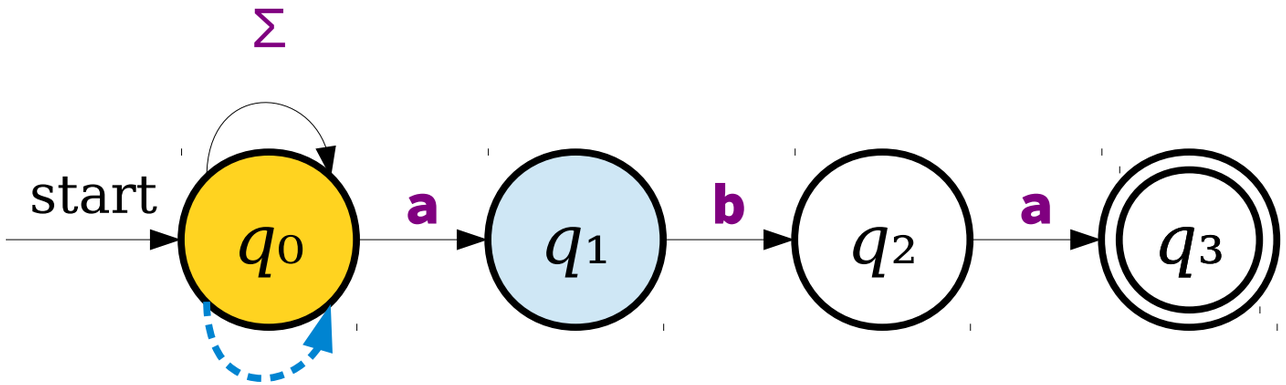

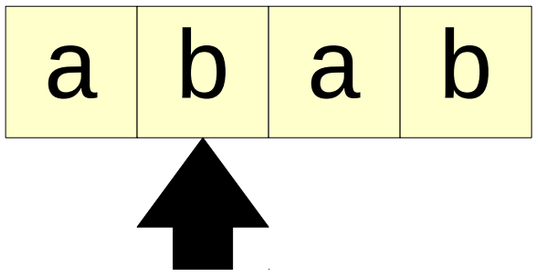

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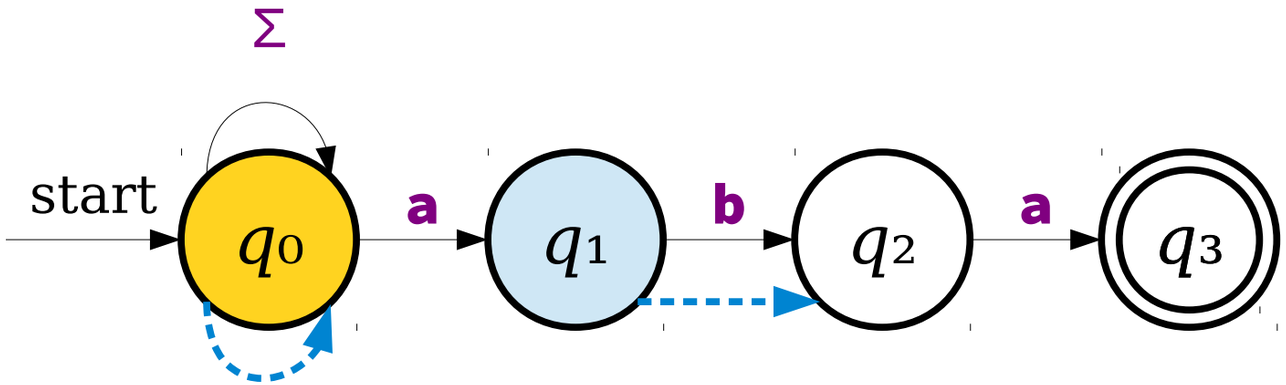

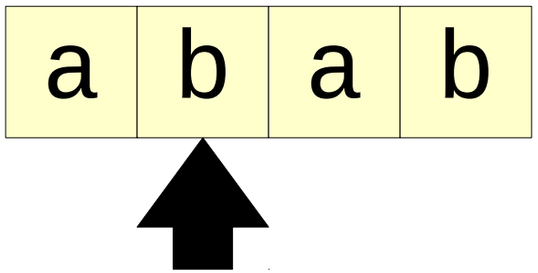

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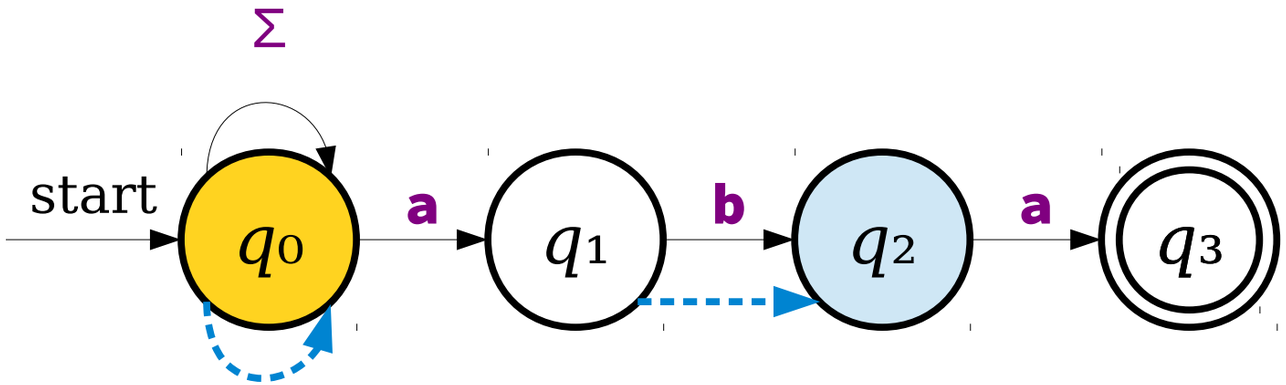

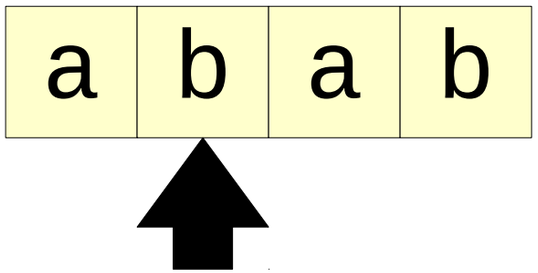

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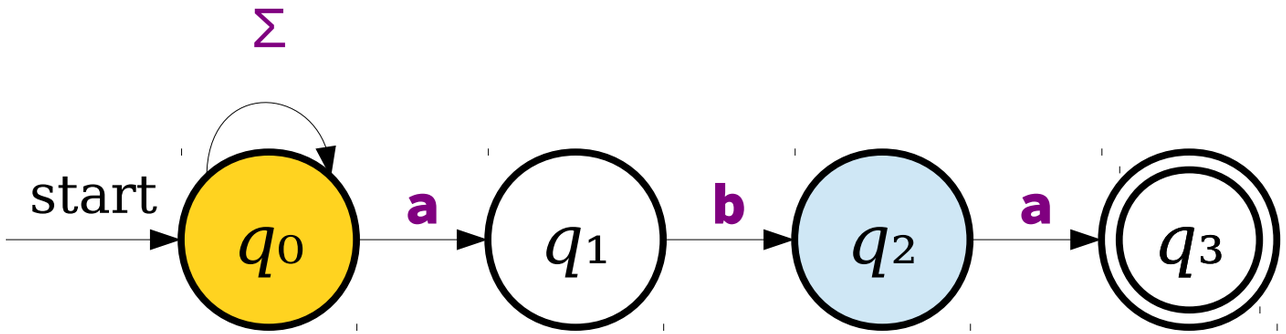

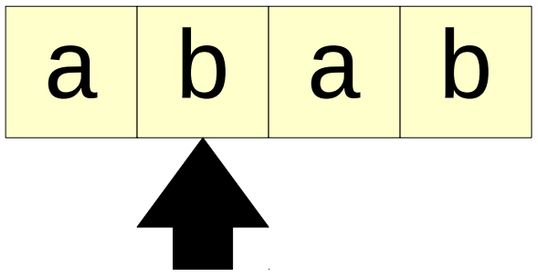

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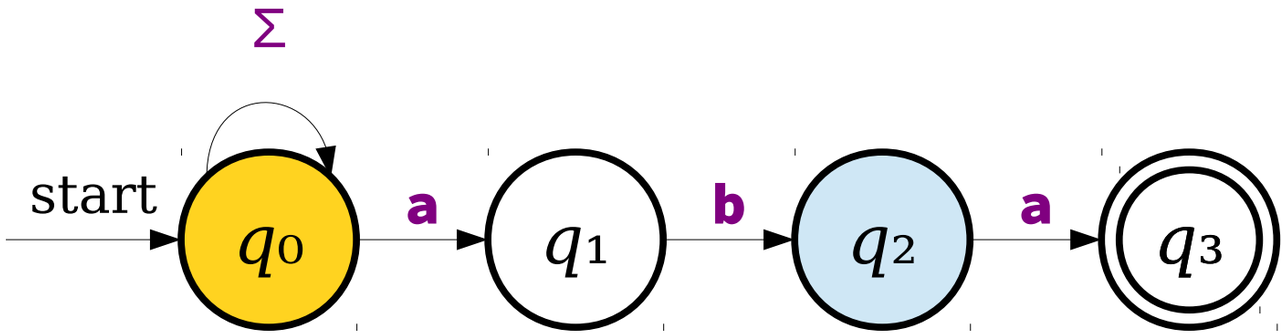

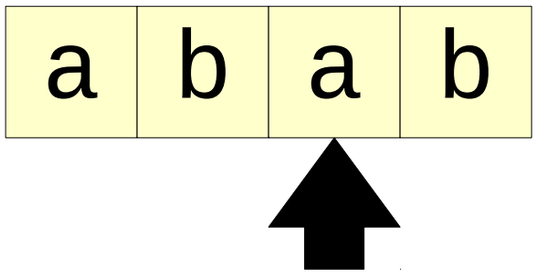

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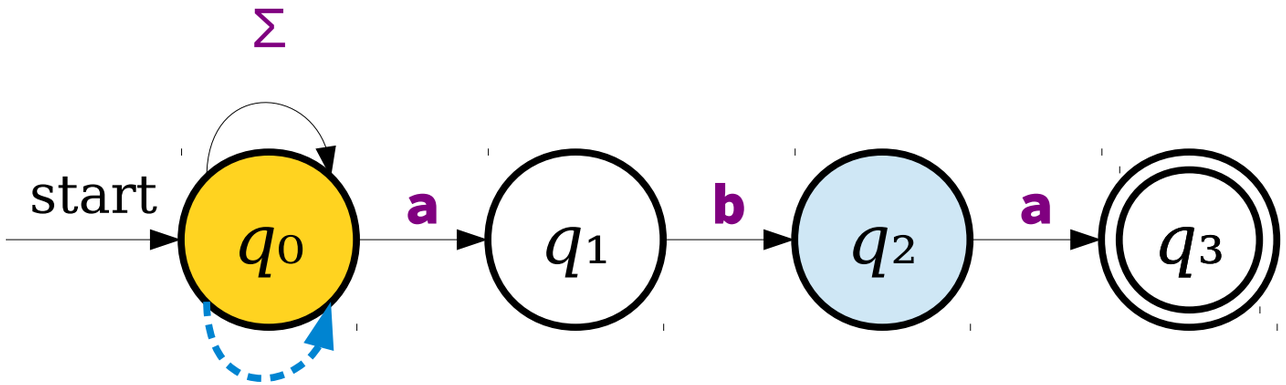

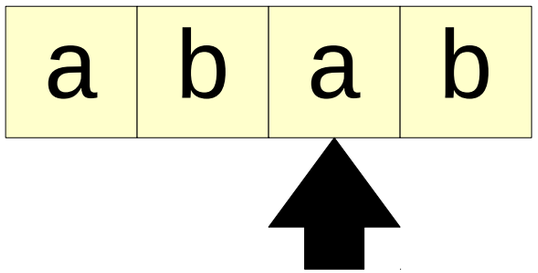

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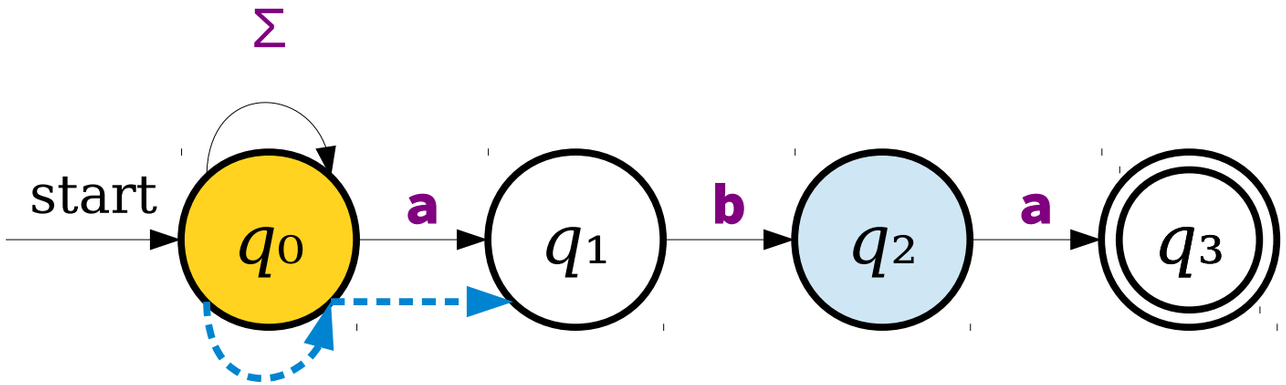

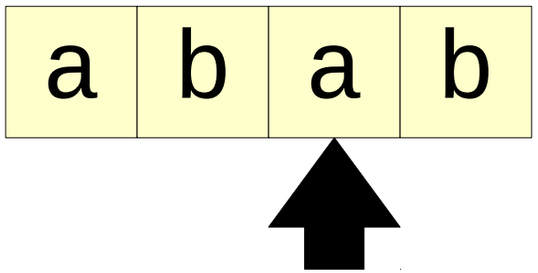

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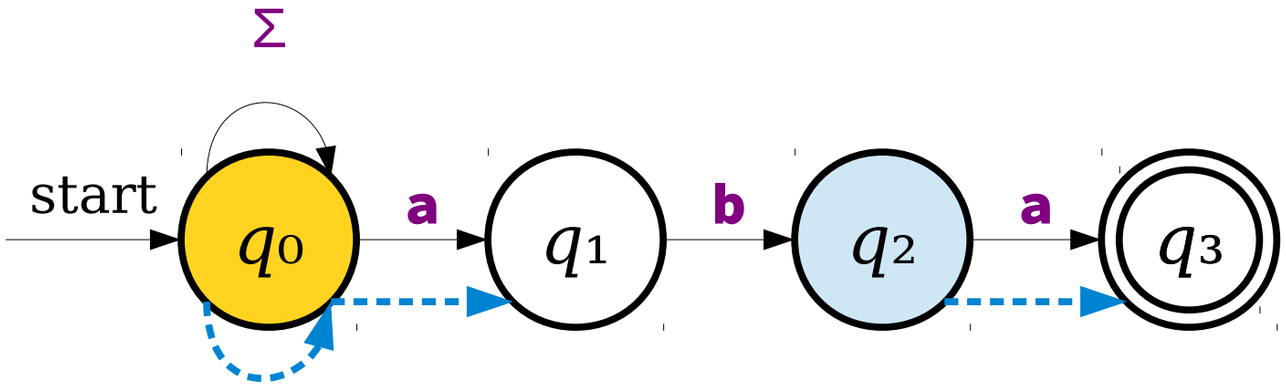

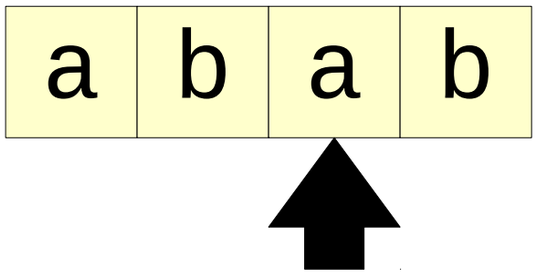

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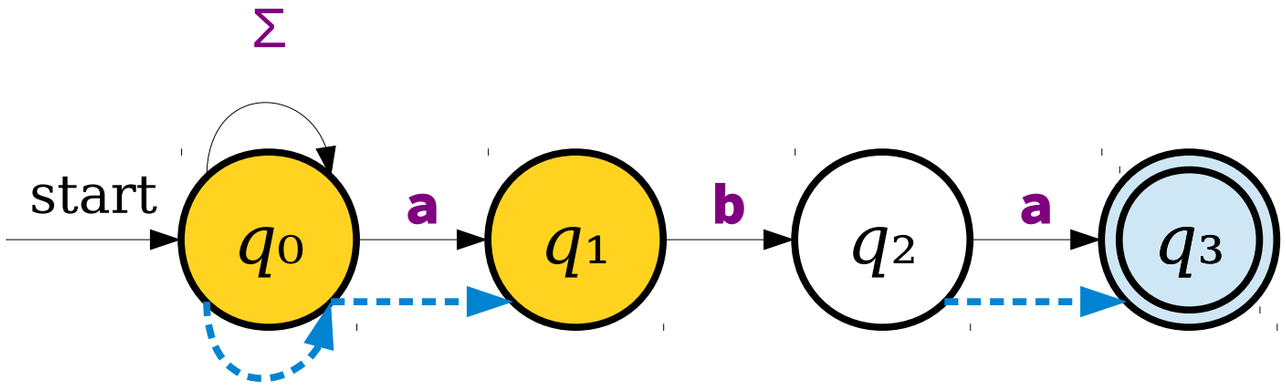

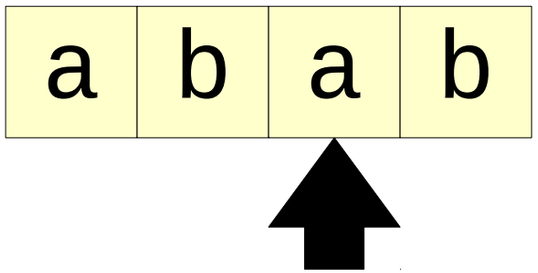

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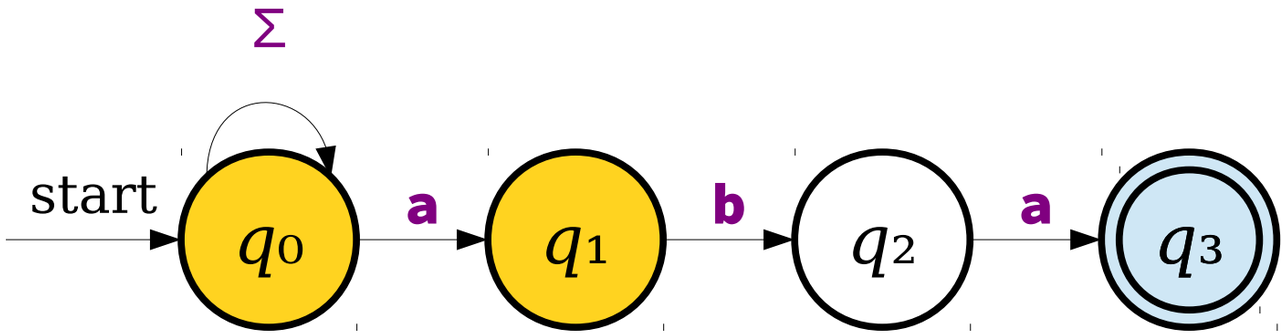

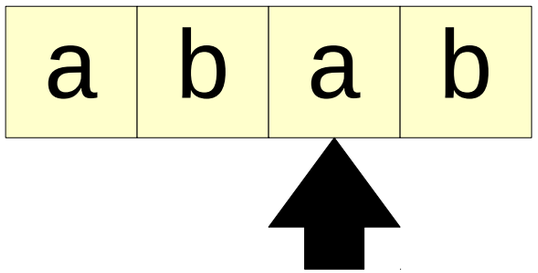

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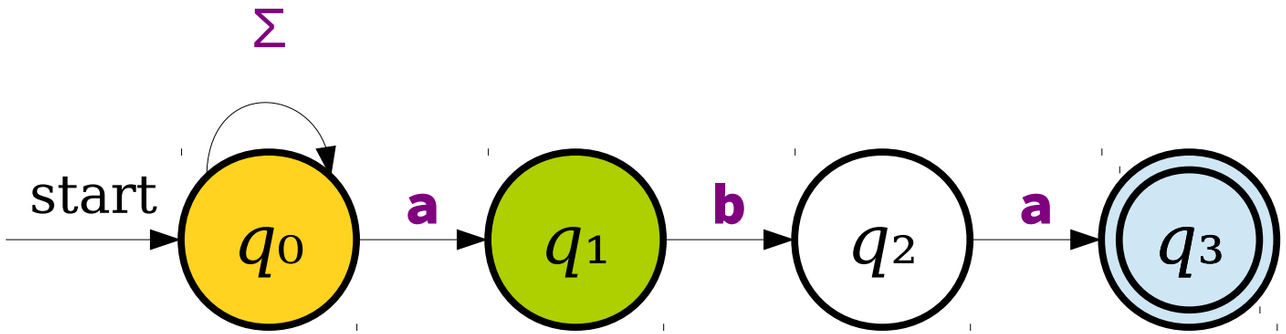

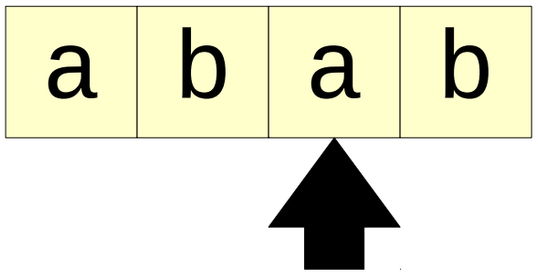

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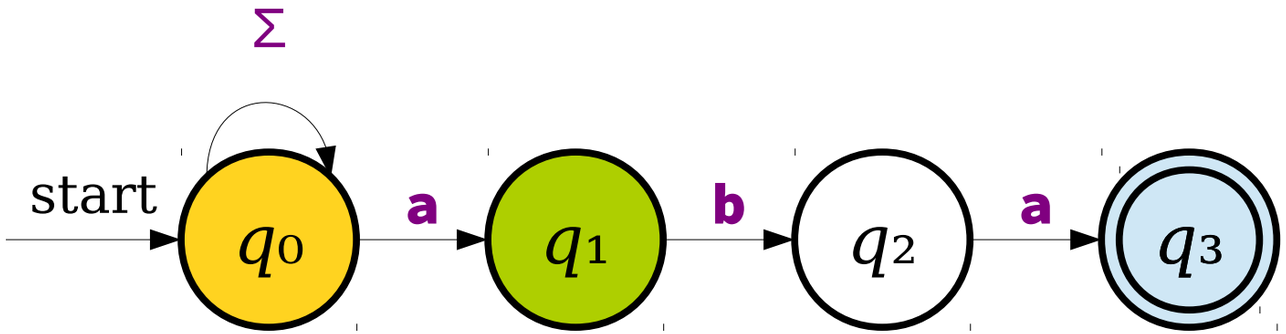

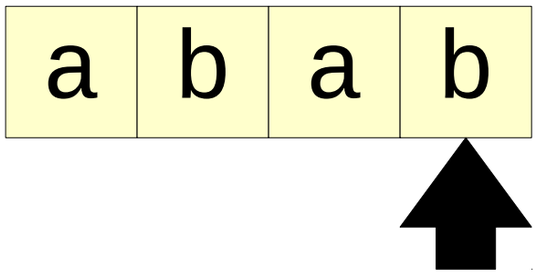

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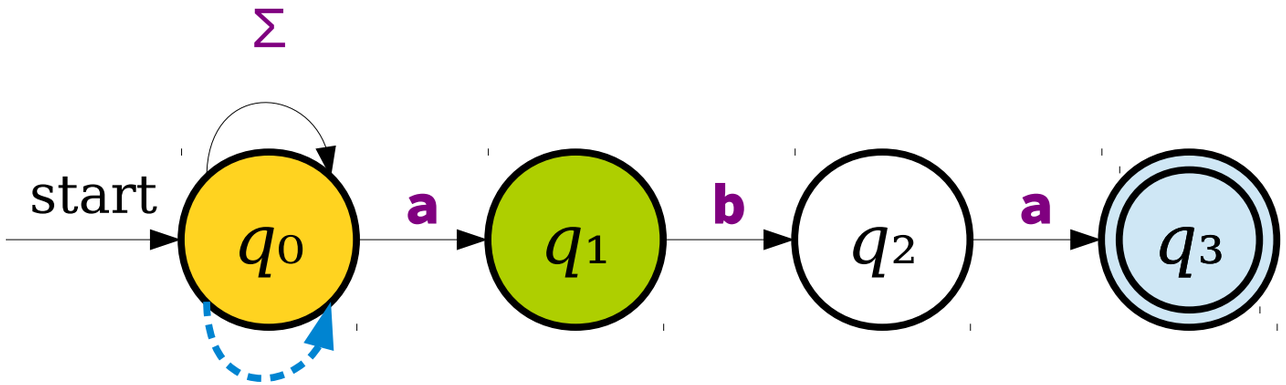

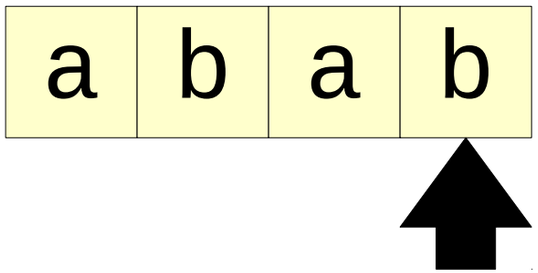

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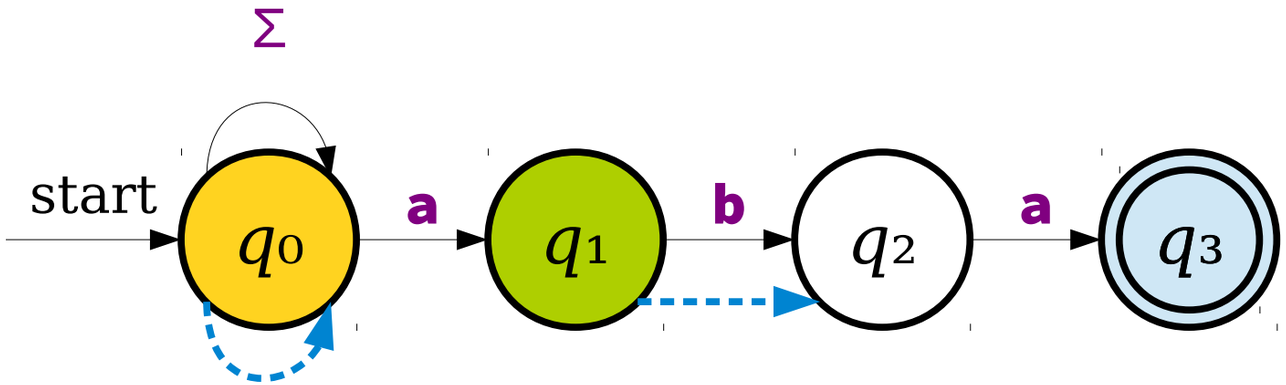

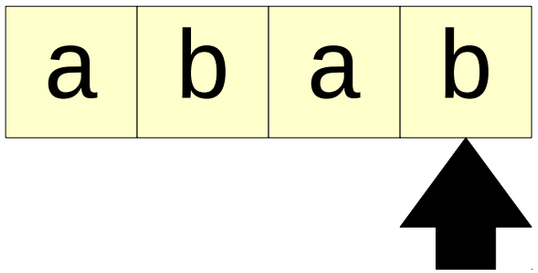

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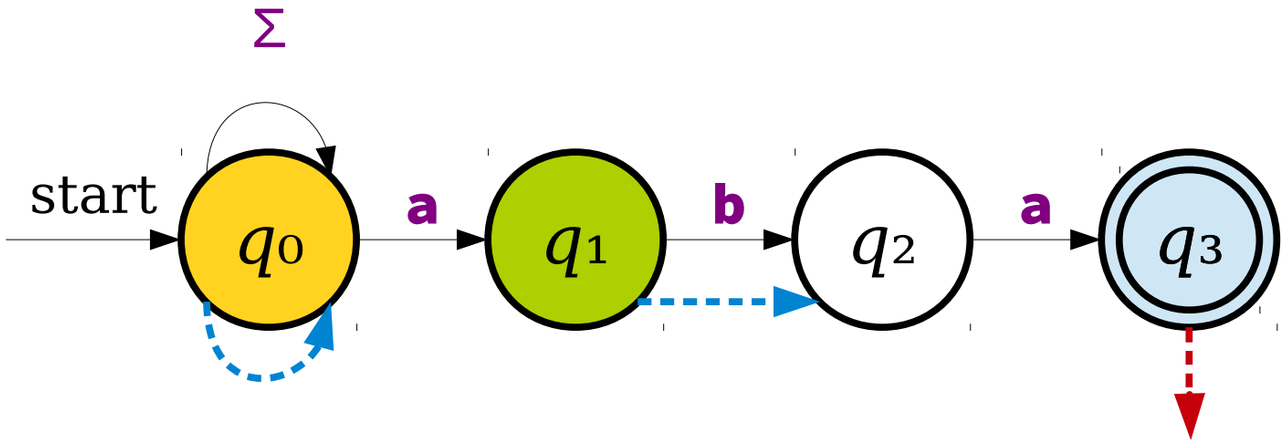

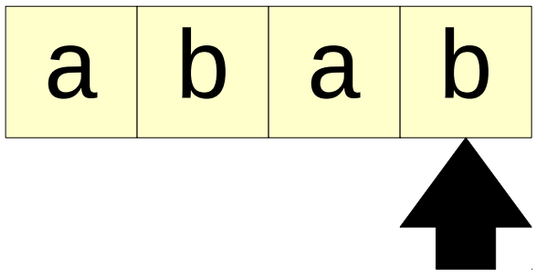

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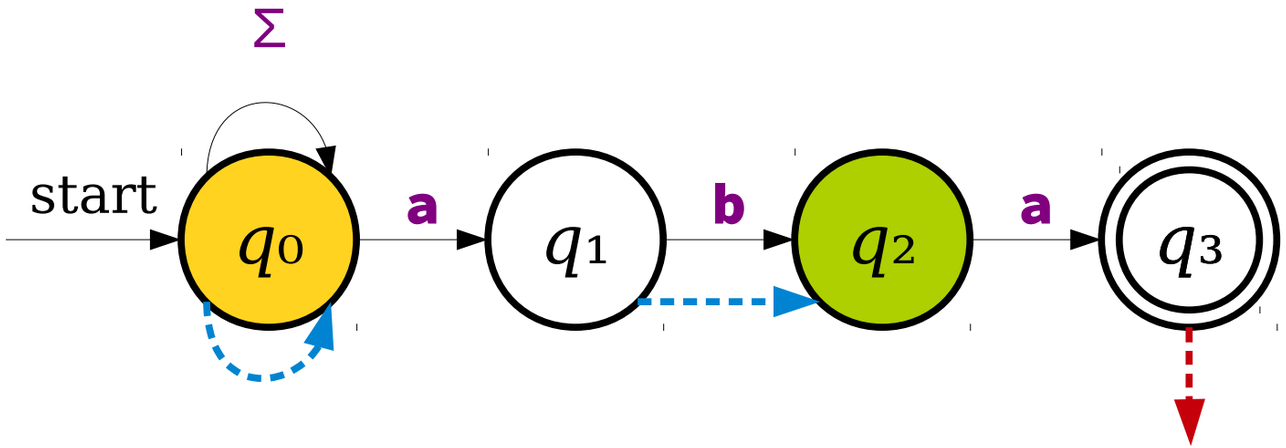

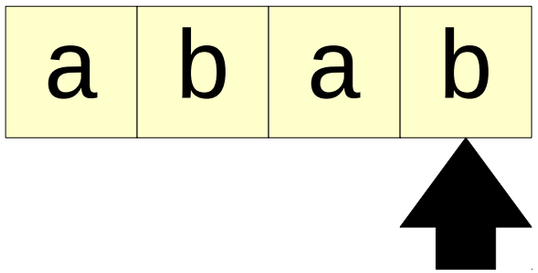

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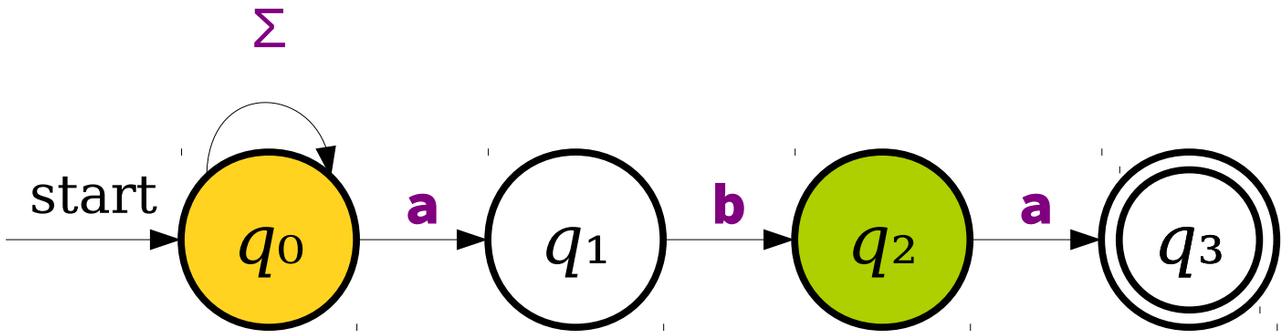

a	b	a	b

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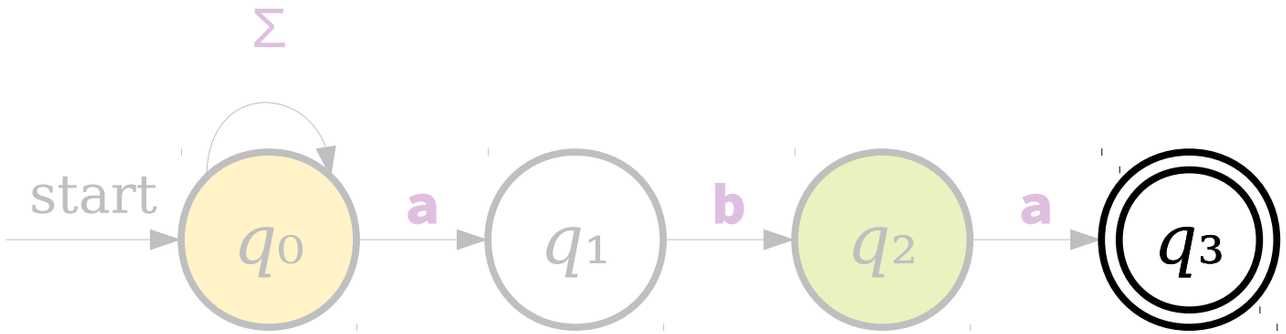

a	b	a	b

We’re not in any accepting state, so no possible path accepts.

Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- (Here's a rigorous explanation about how this works; read this on your own time).
  - Start off in the set of all states formed by taking the start state and including each state that can be reached by zero or more ε-transitions.
  - When you read a symbol **a** in a set of states *S*:
    - Form the set *S'* of states that can be reached by following a single **a** transition from some state in *S*.
    - Your new set of states is the set of states in *S'*, plus the states reachable from *S'* by following zero or more ε-transitions.

Designing NFAs

Designing NFAs

- ***Embrace the nondeterminism!***
- Good model: ***Guess-and-check***:
  - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
  - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

Guess-and-Check

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ ends in } \texttt{010} \text{ or } \texttt{101} \}$

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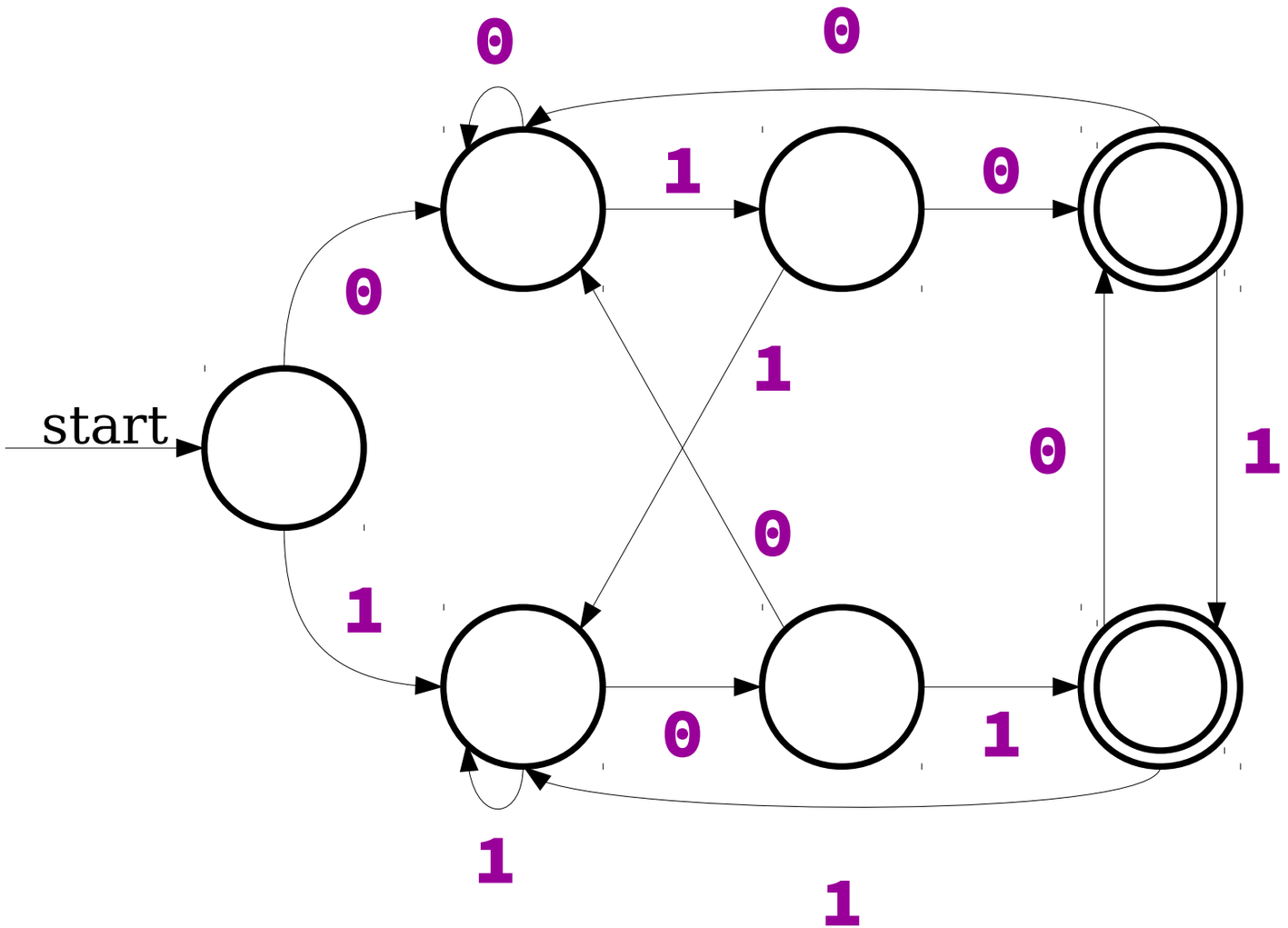

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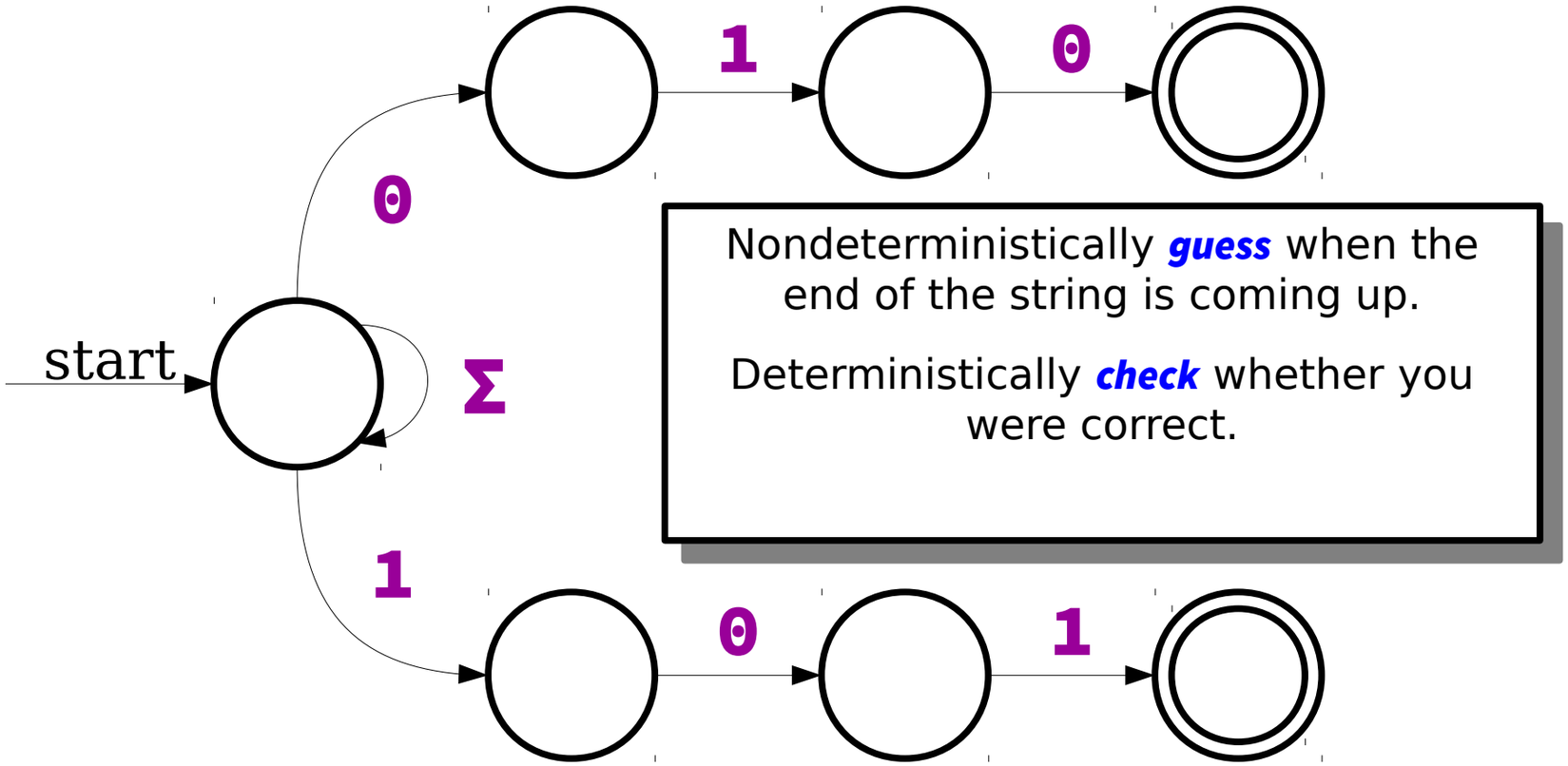

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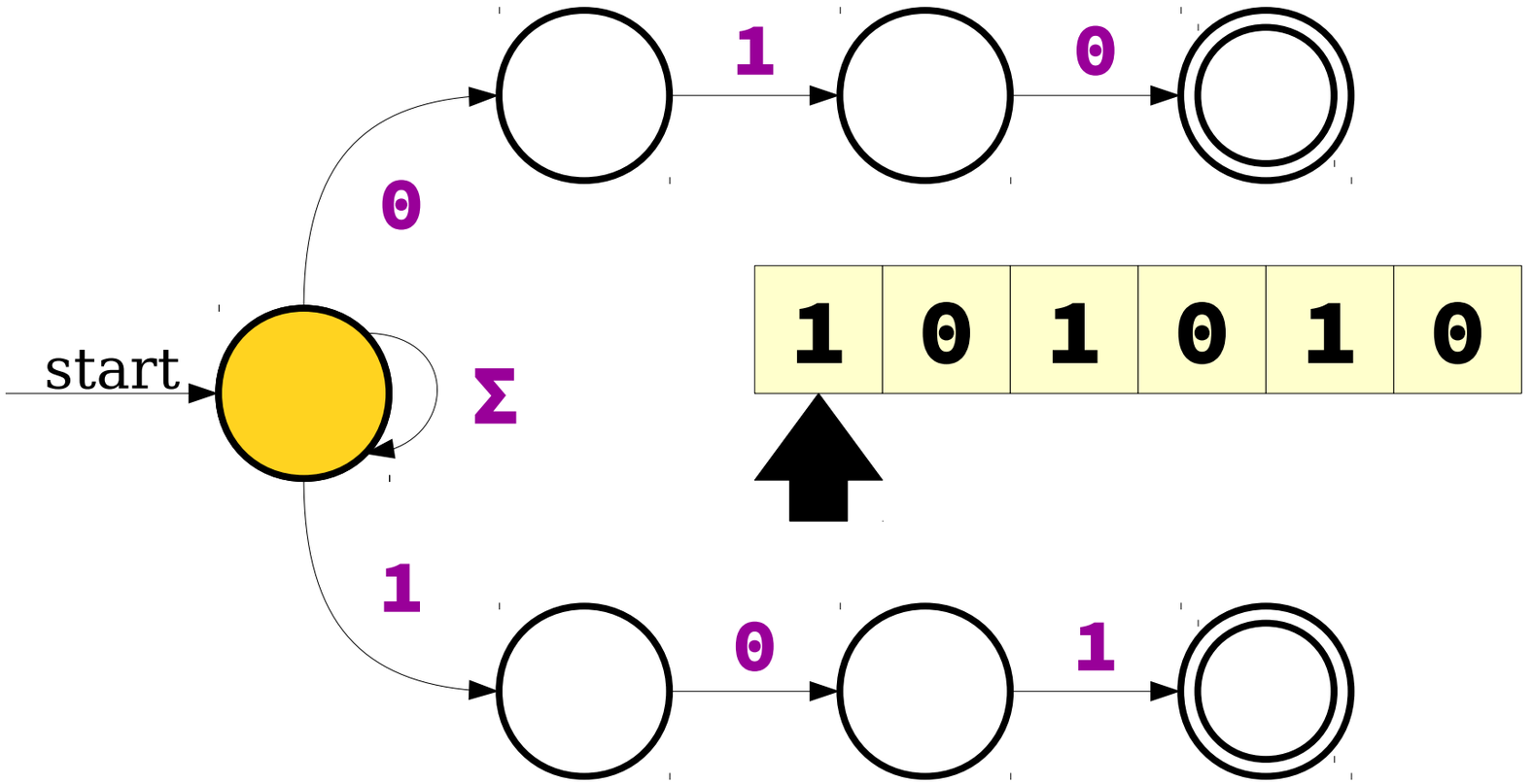

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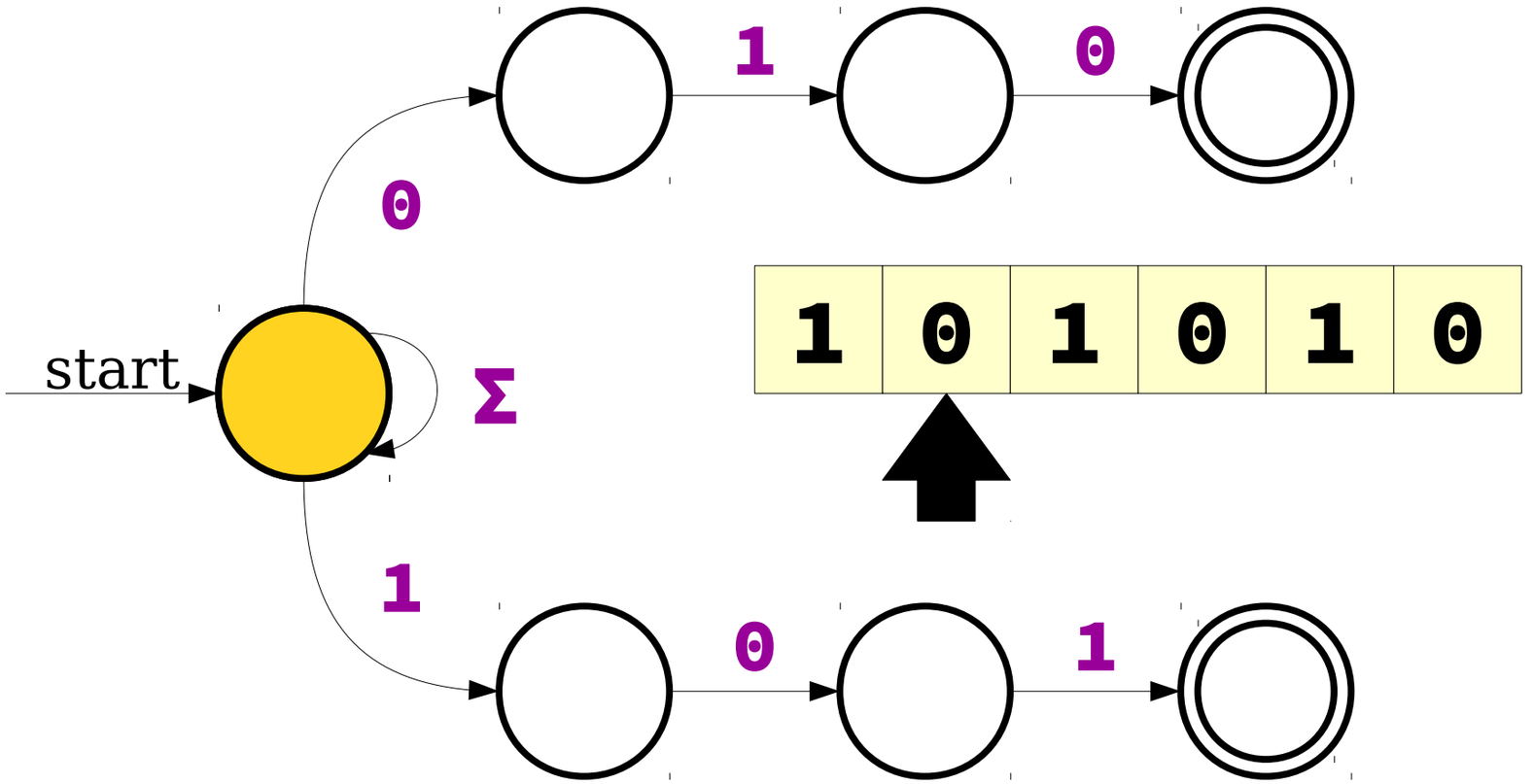

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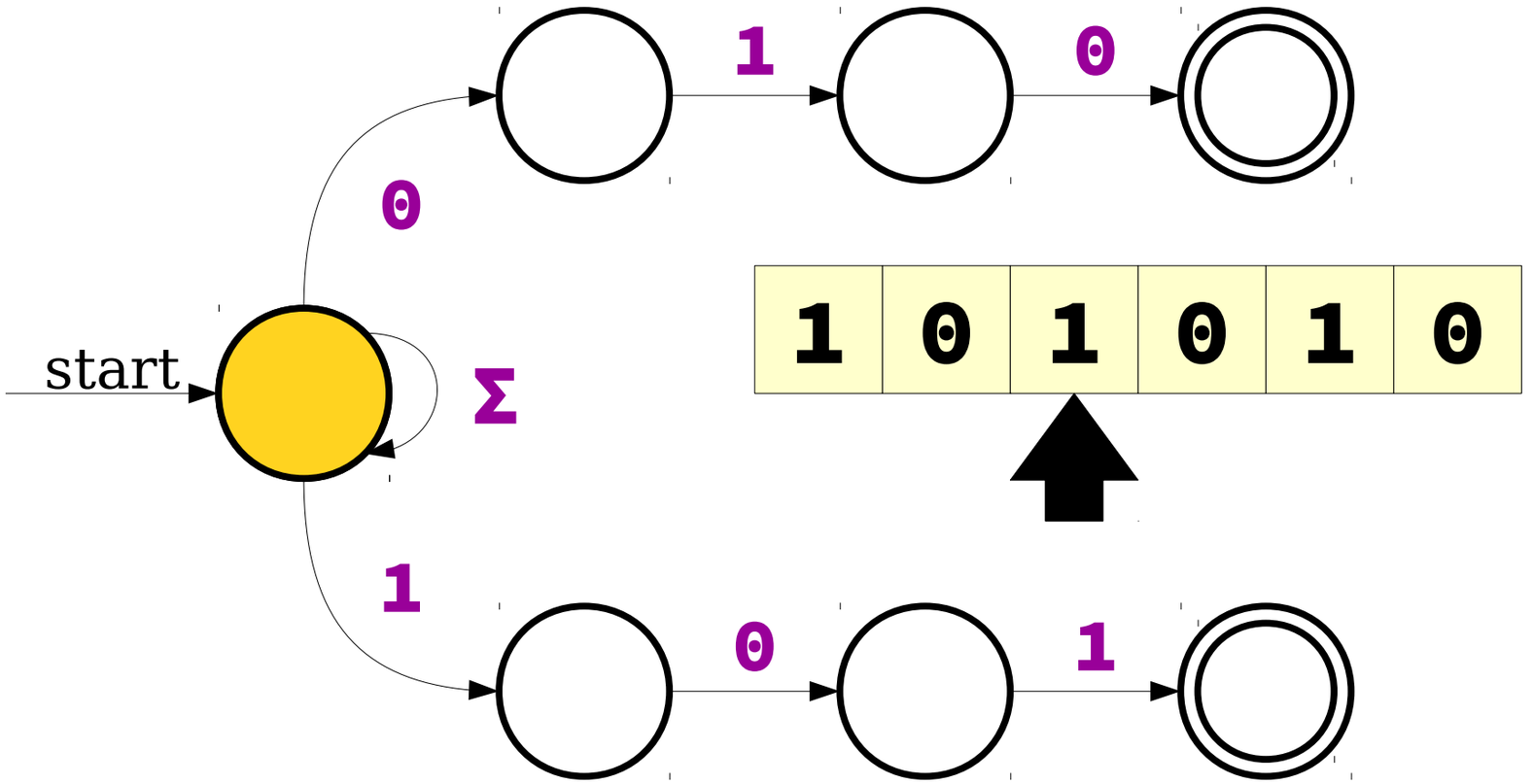

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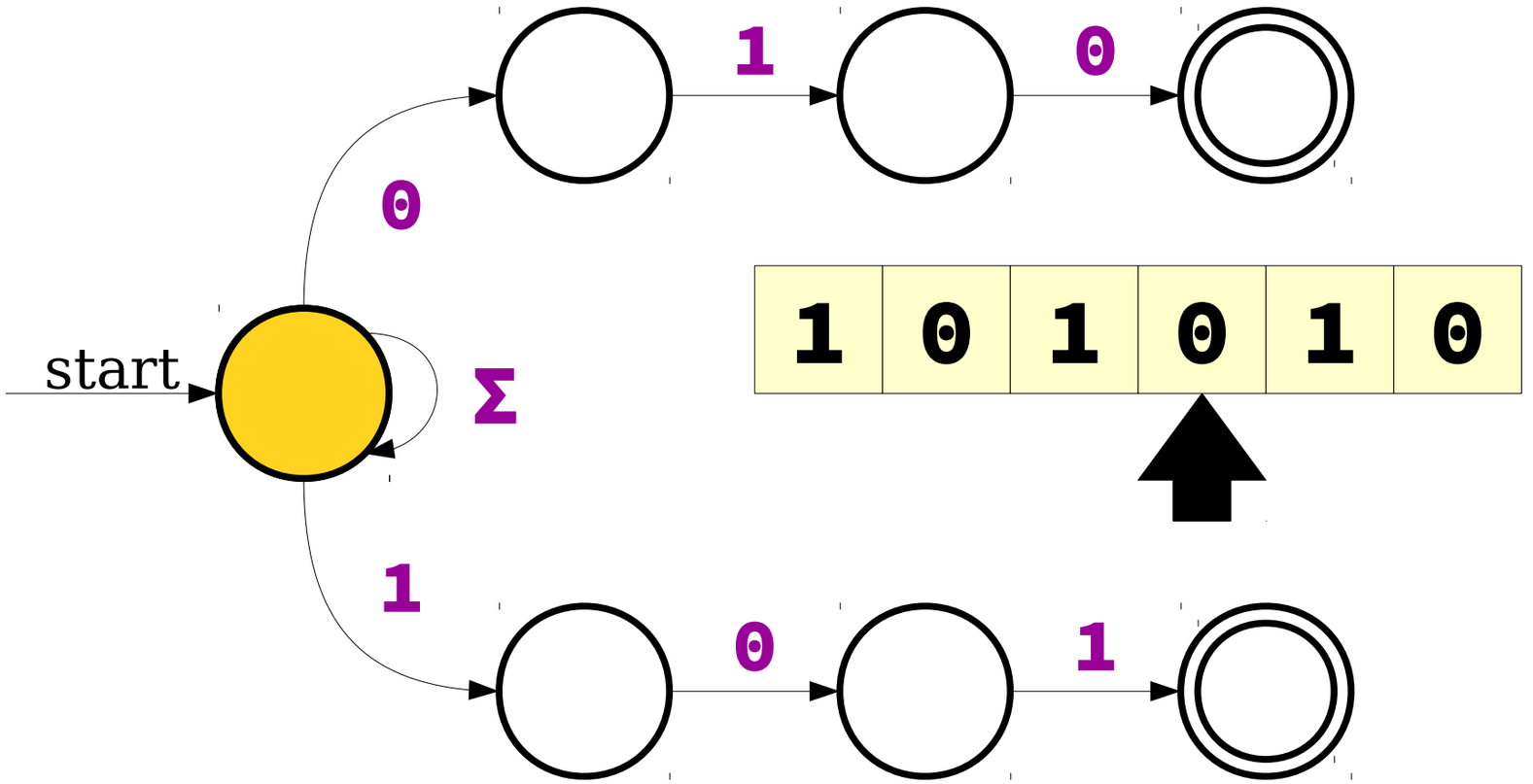

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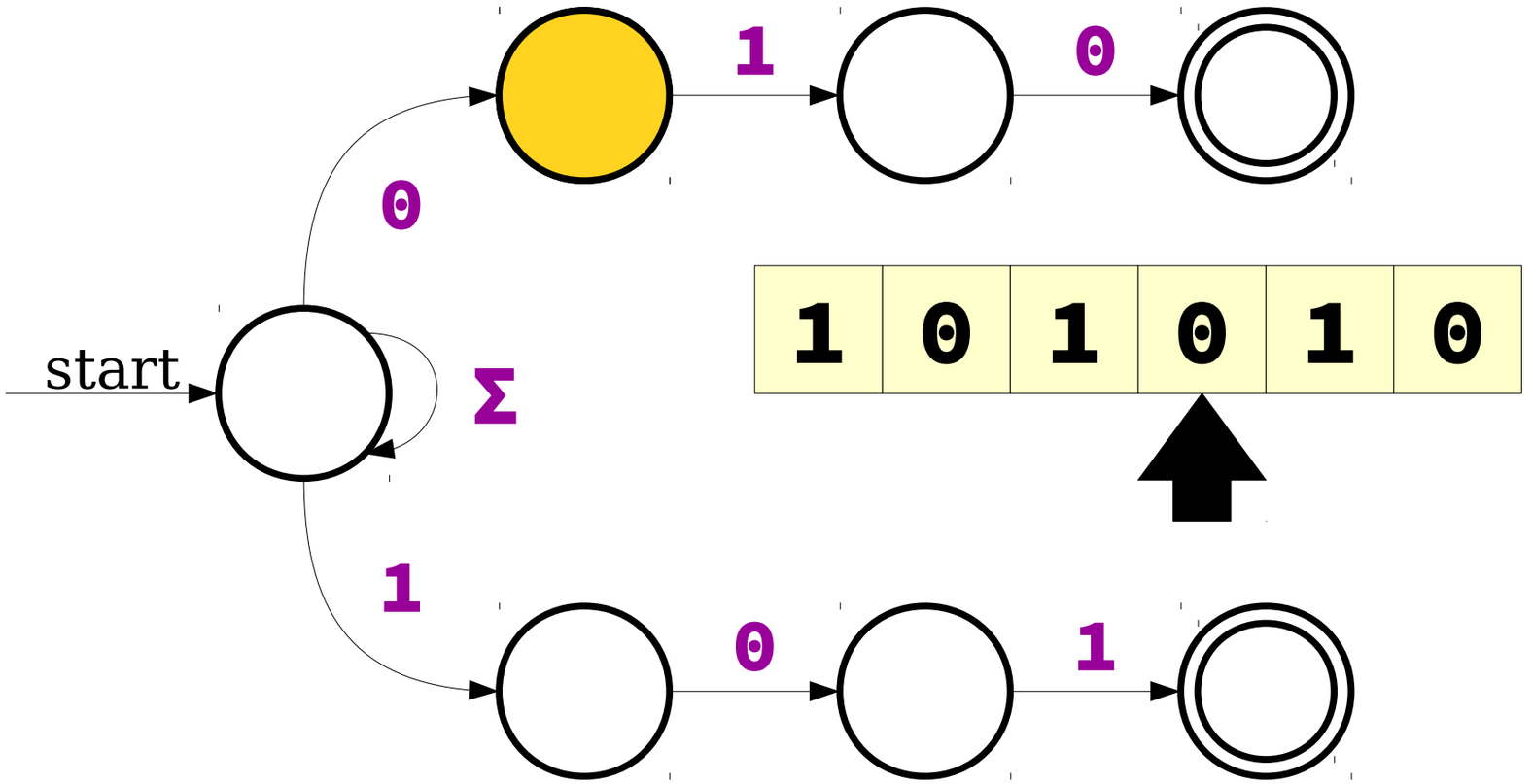

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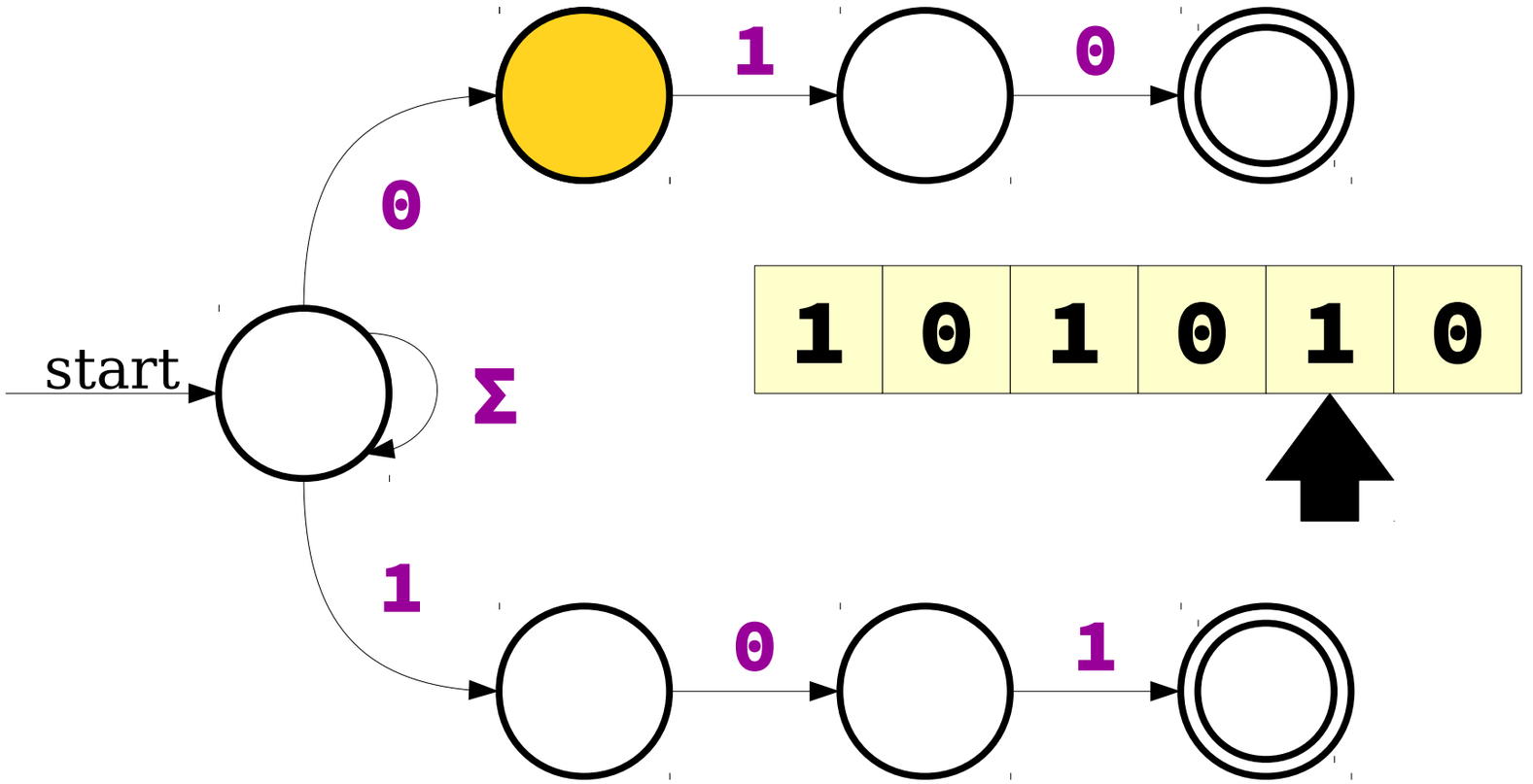

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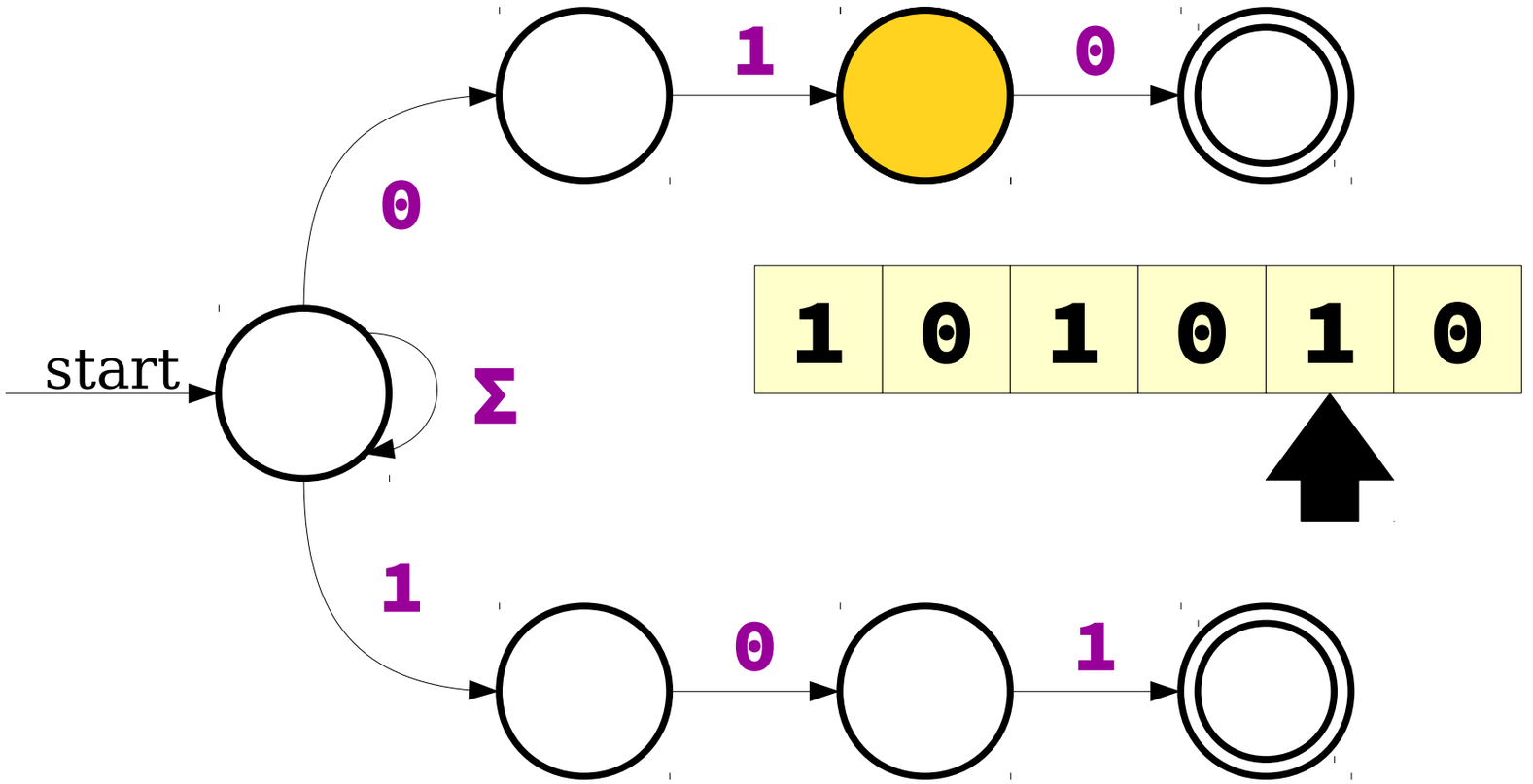

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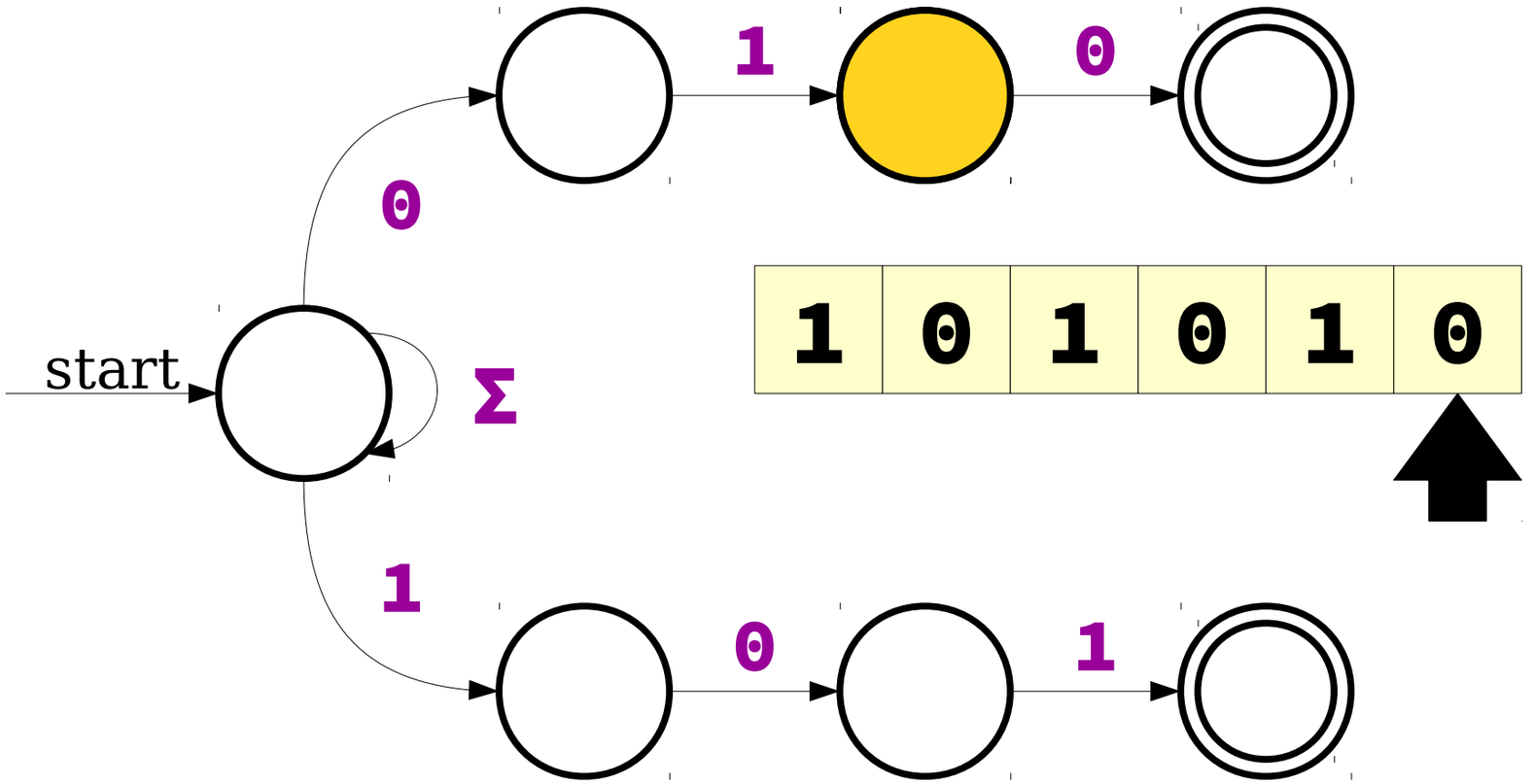

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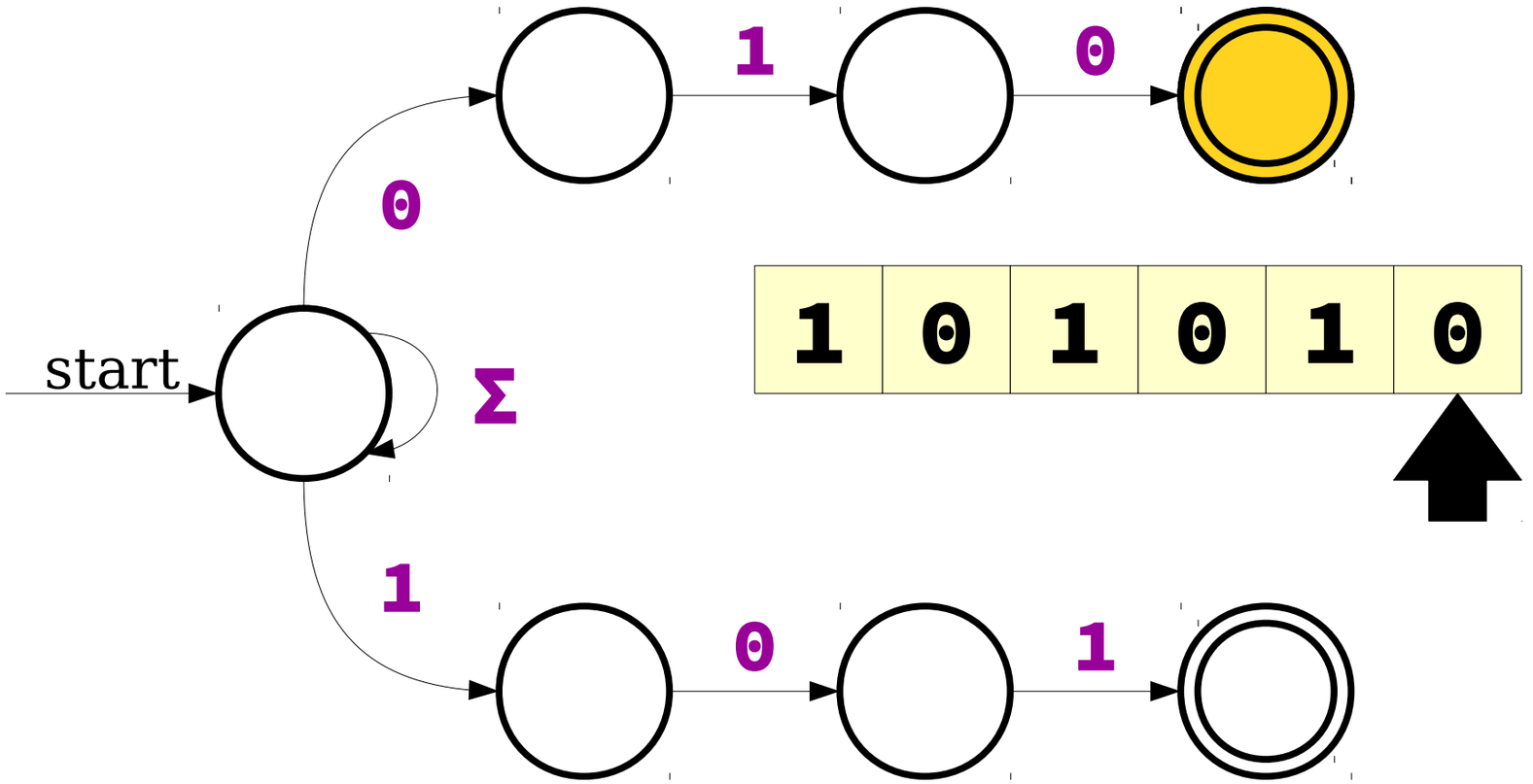

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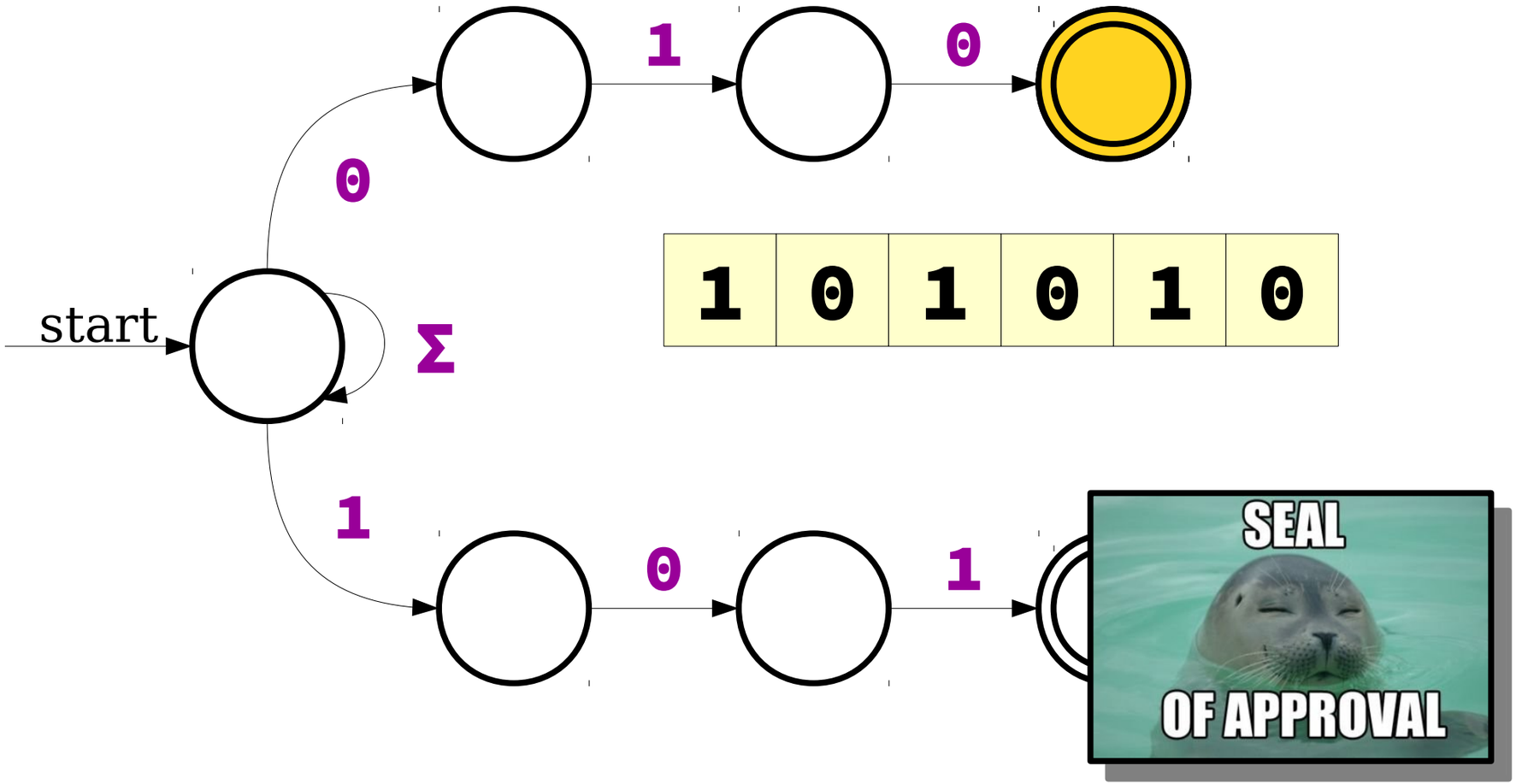

Guess-and-Check

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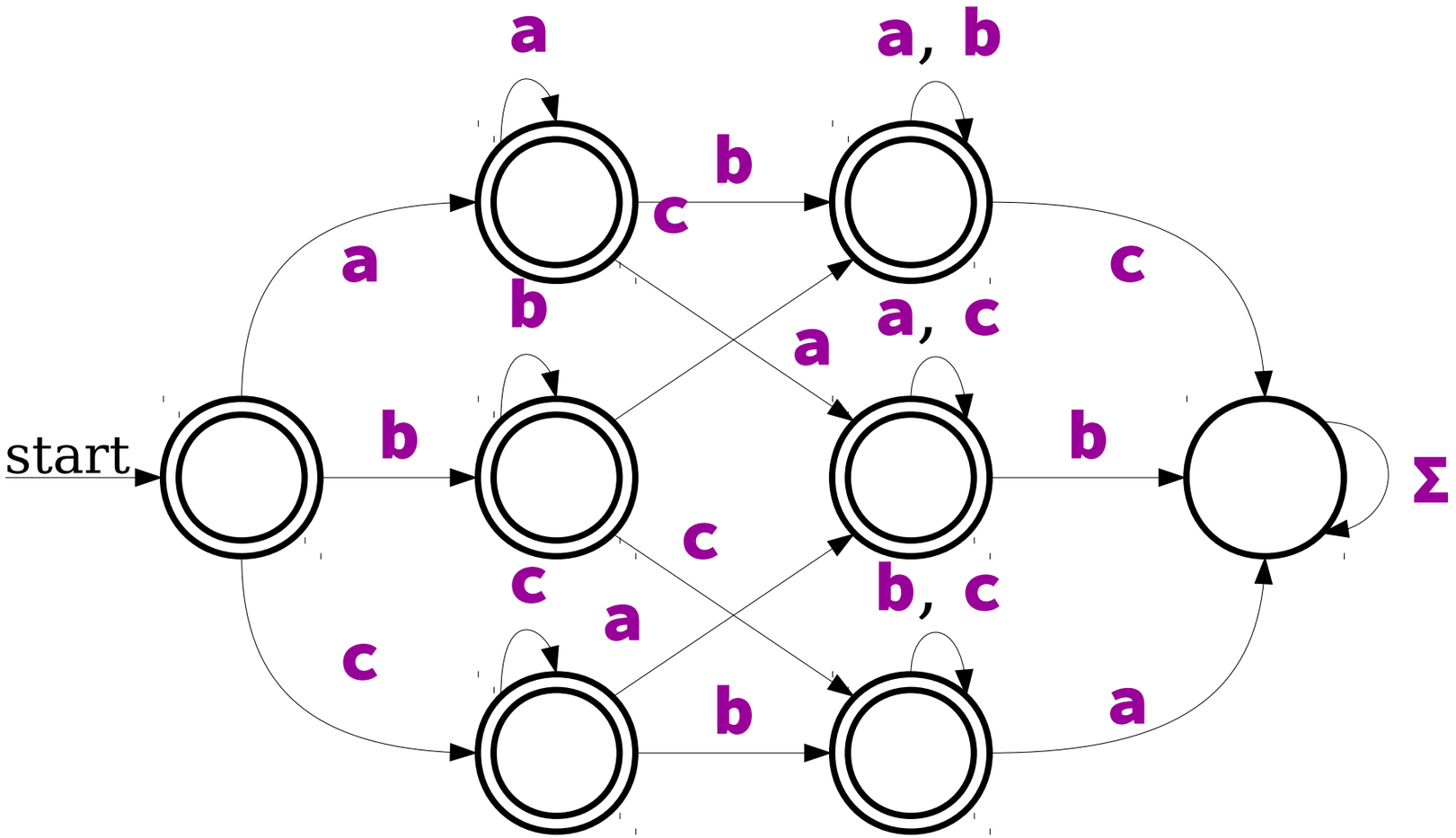

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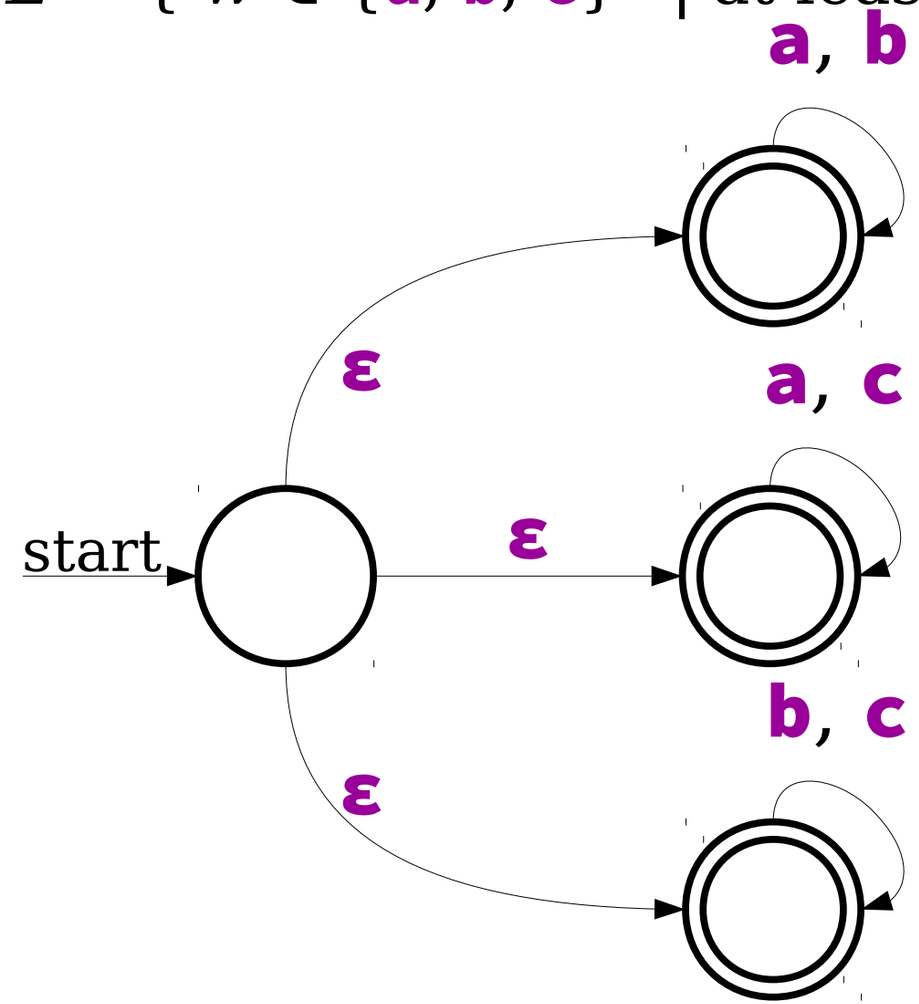

Nondeterministically ***guess*** which character is missing.

Deterministically ***check*** whether that character is indeed missing.

Guess-and-Check

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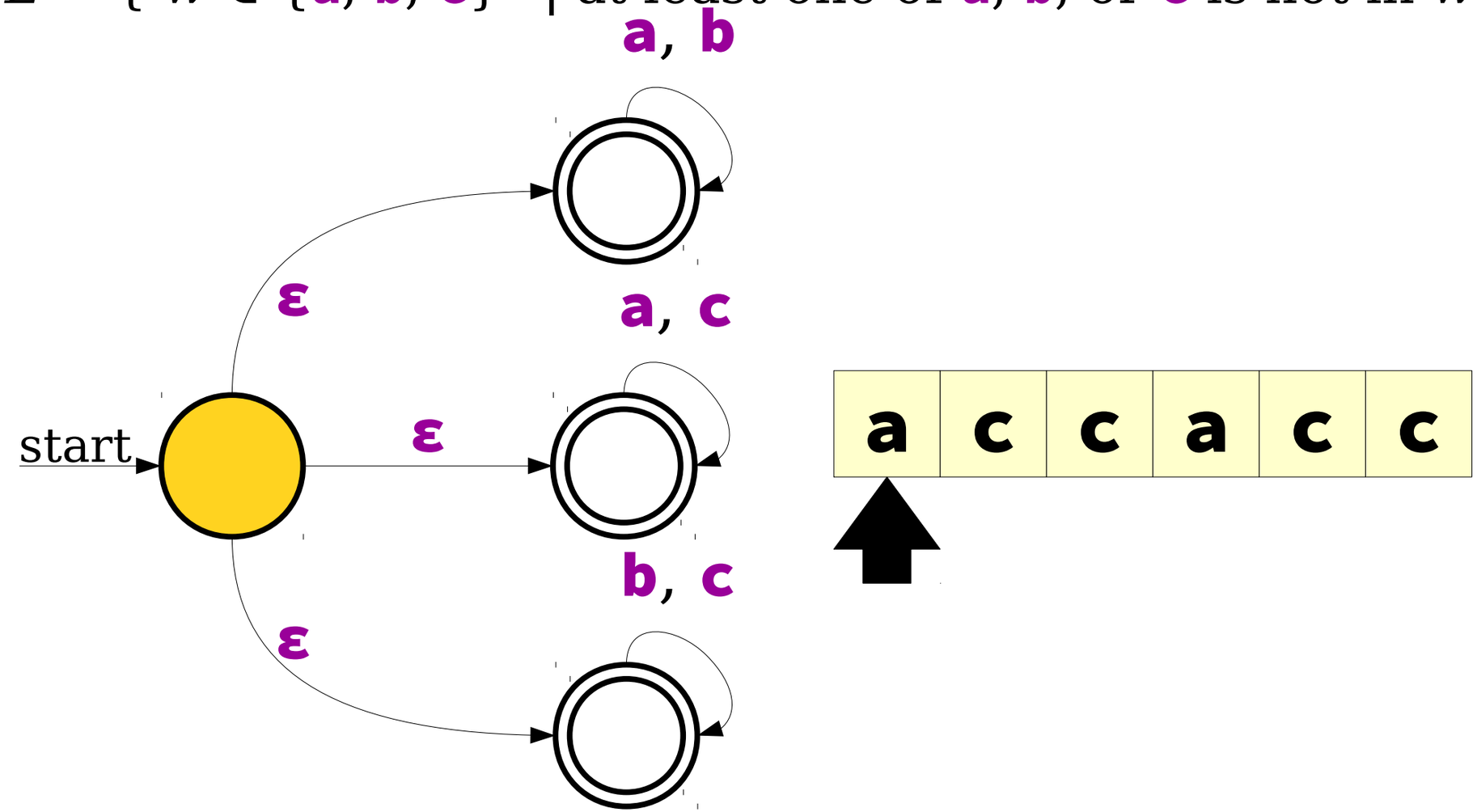

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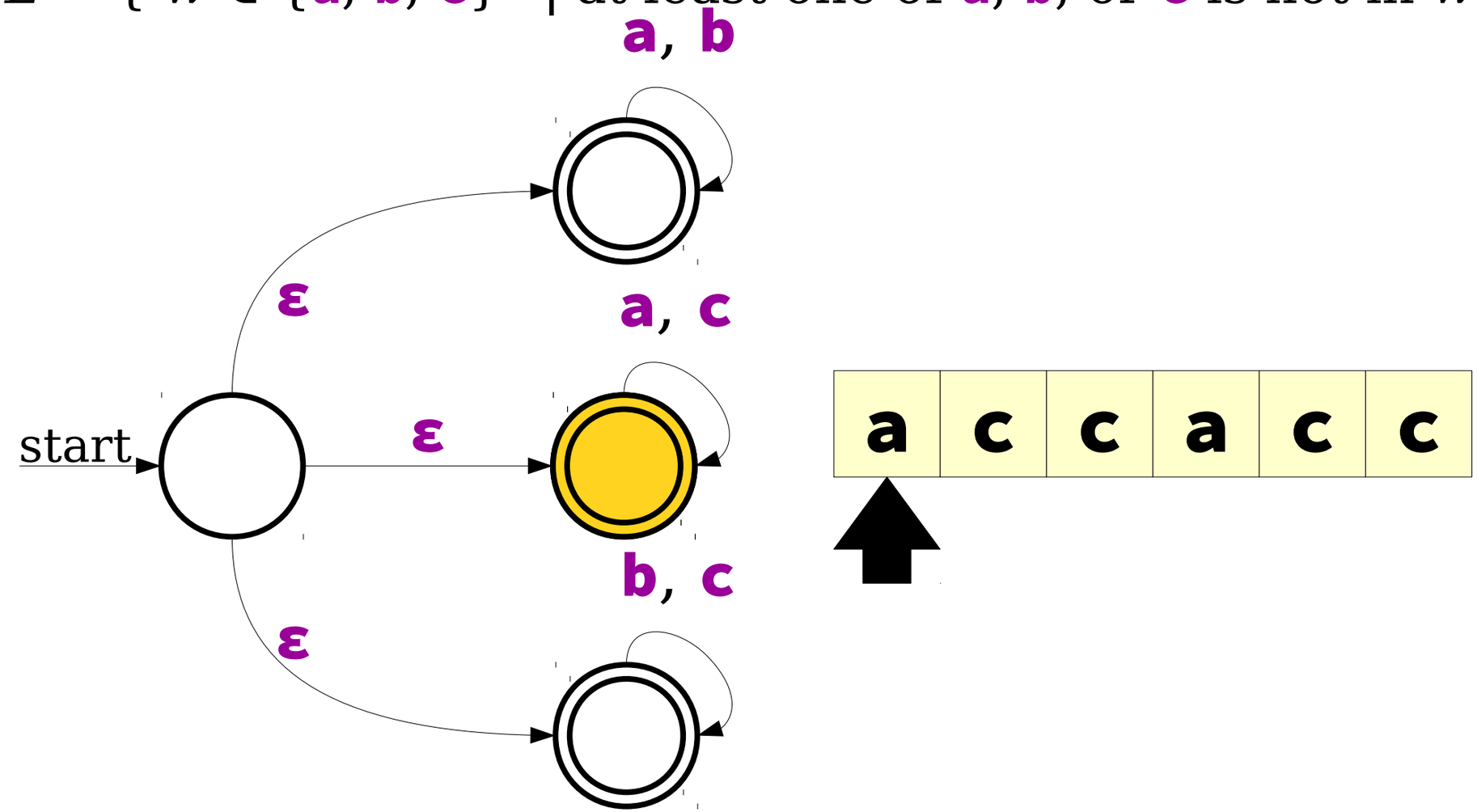

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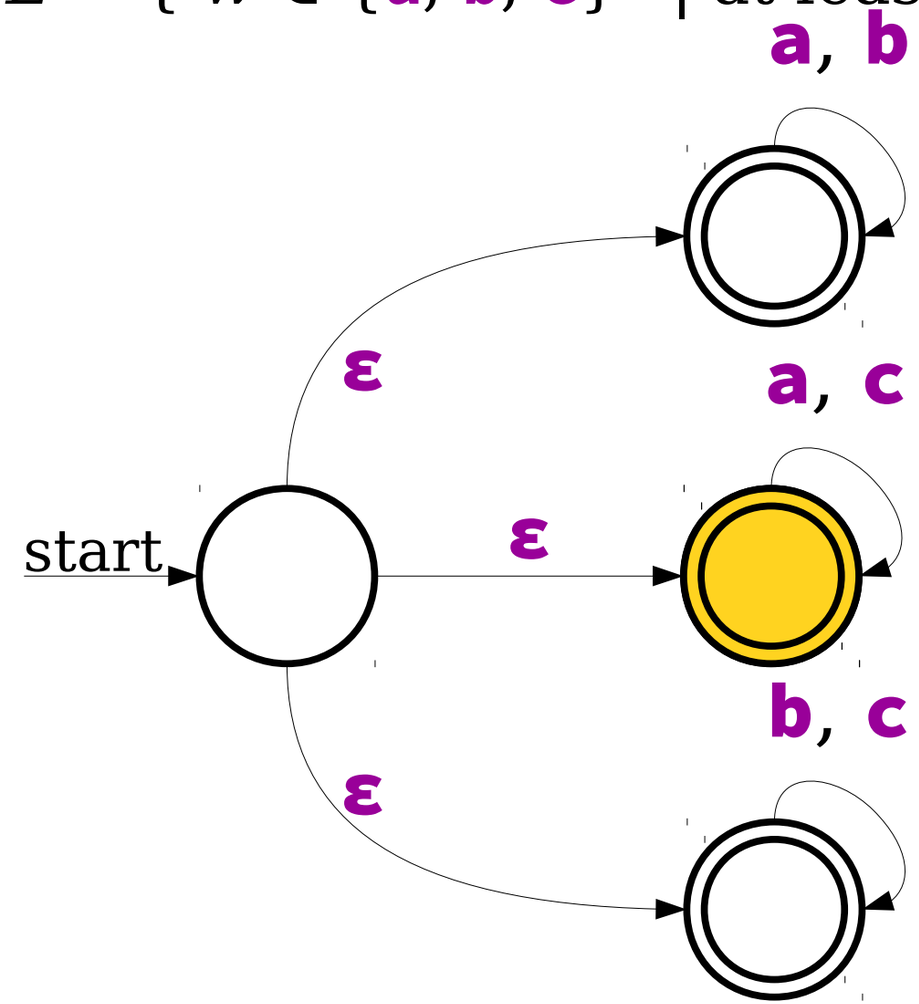

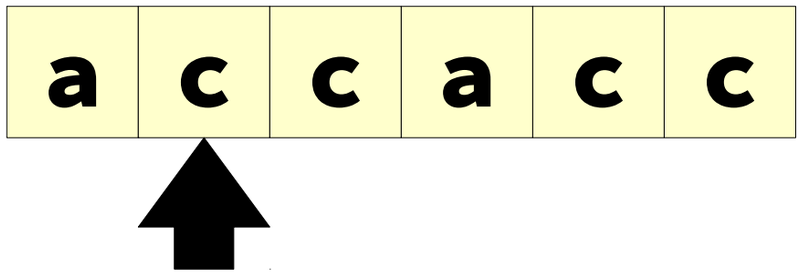

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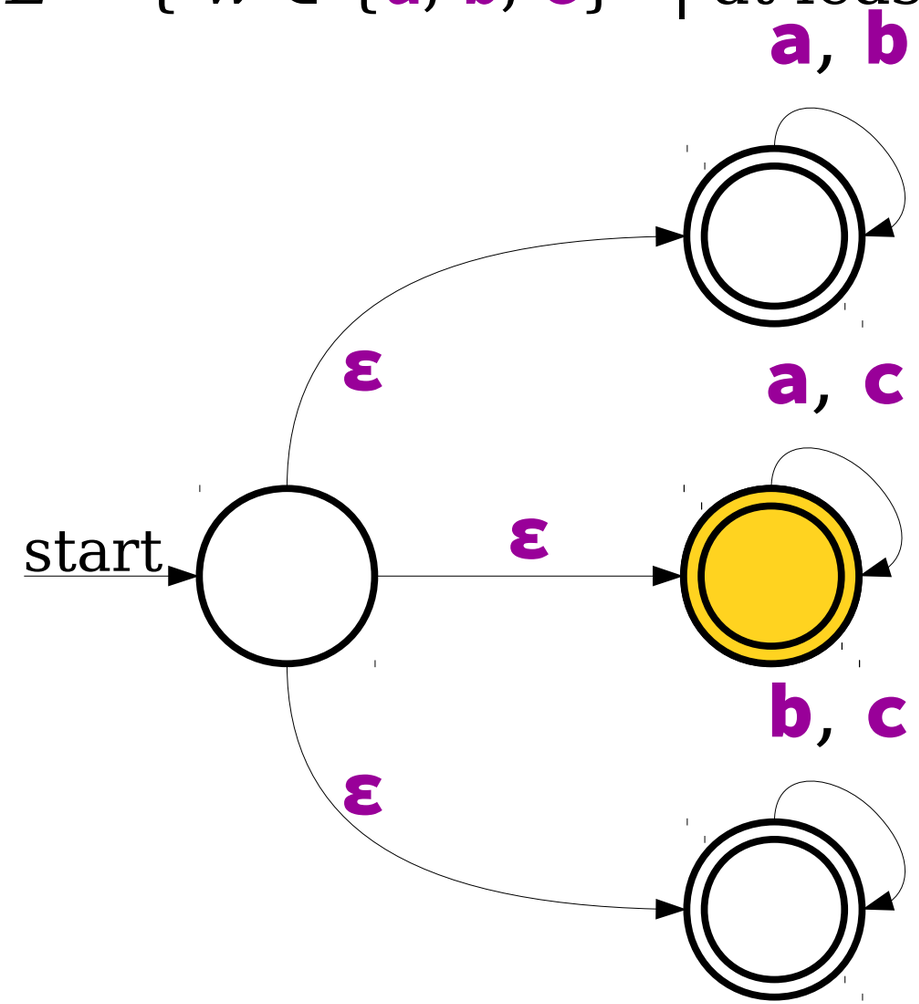

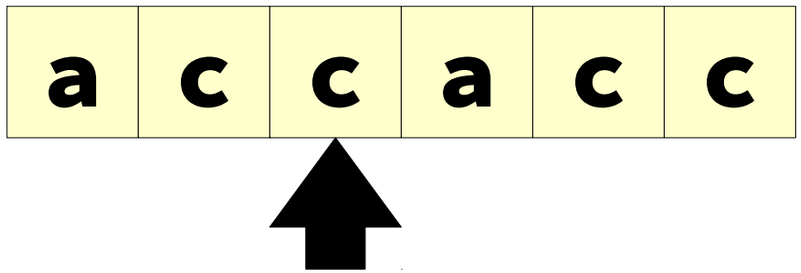

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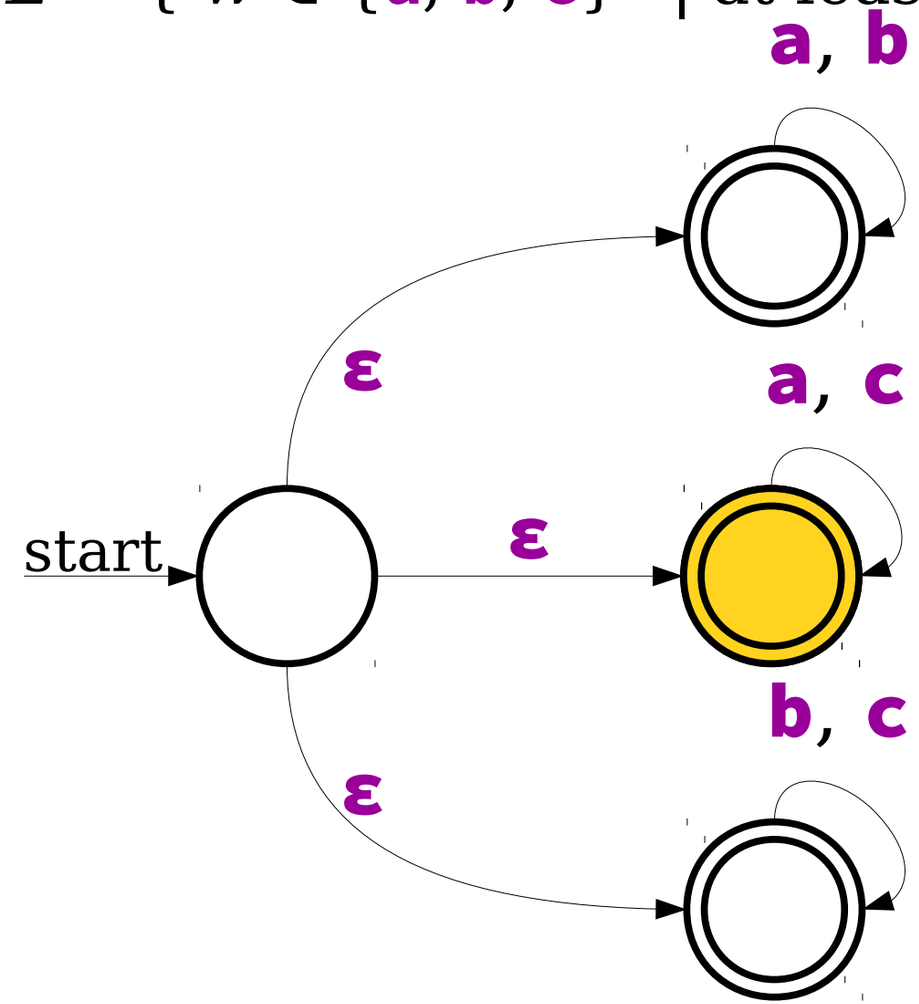

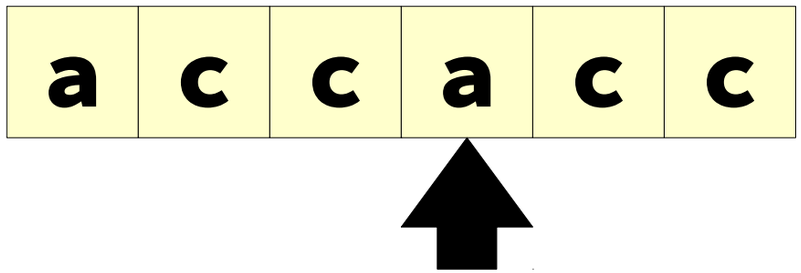

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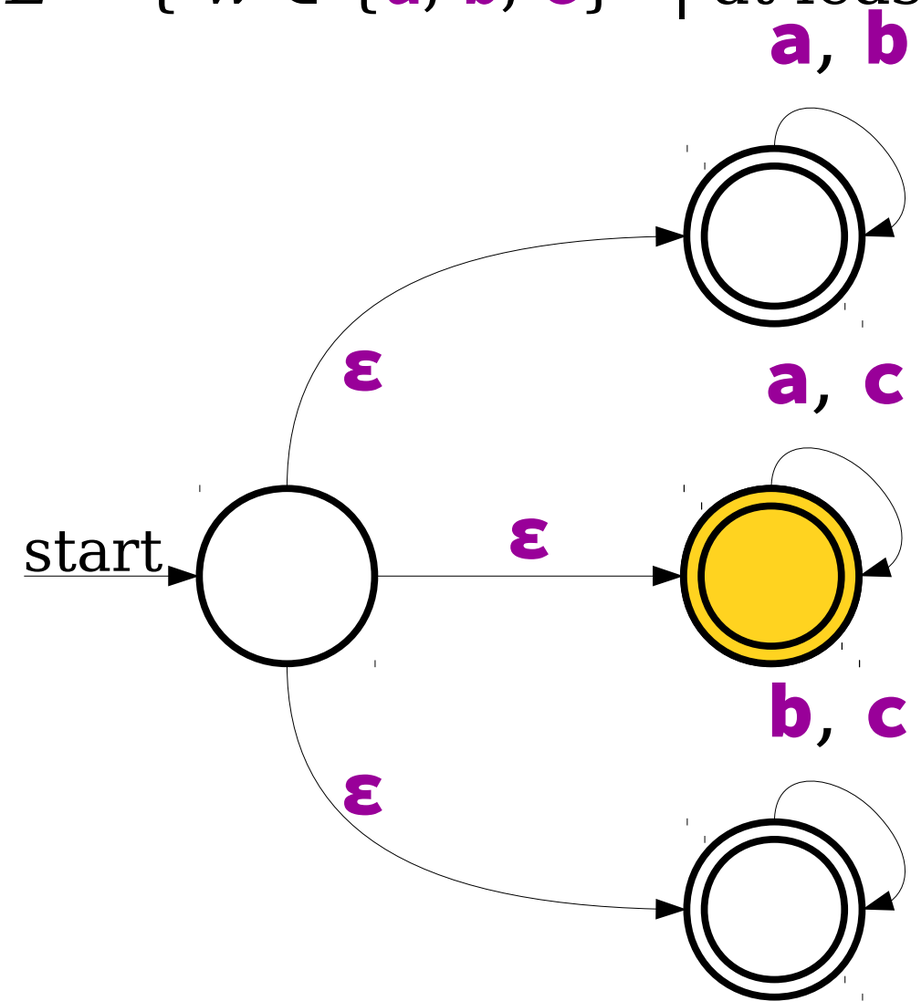

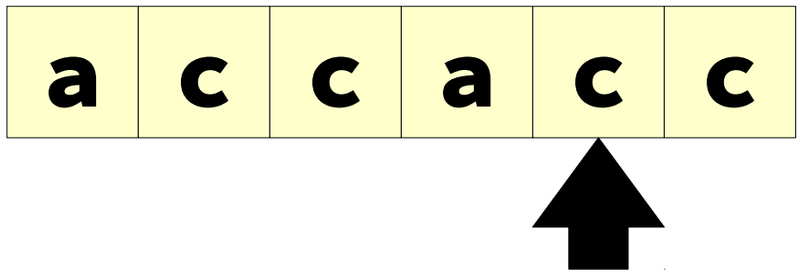

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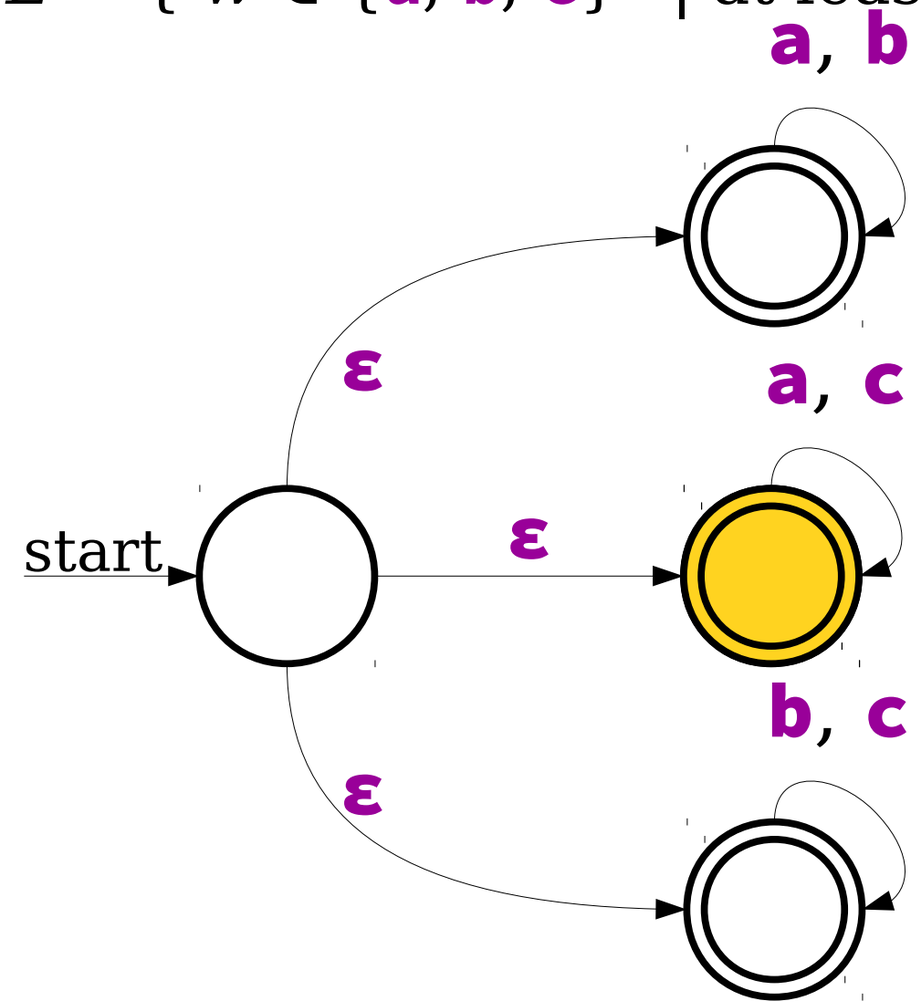

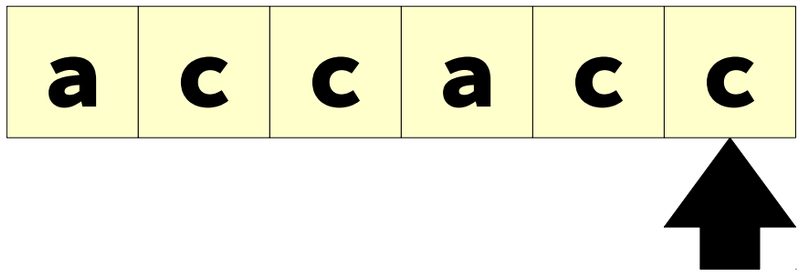

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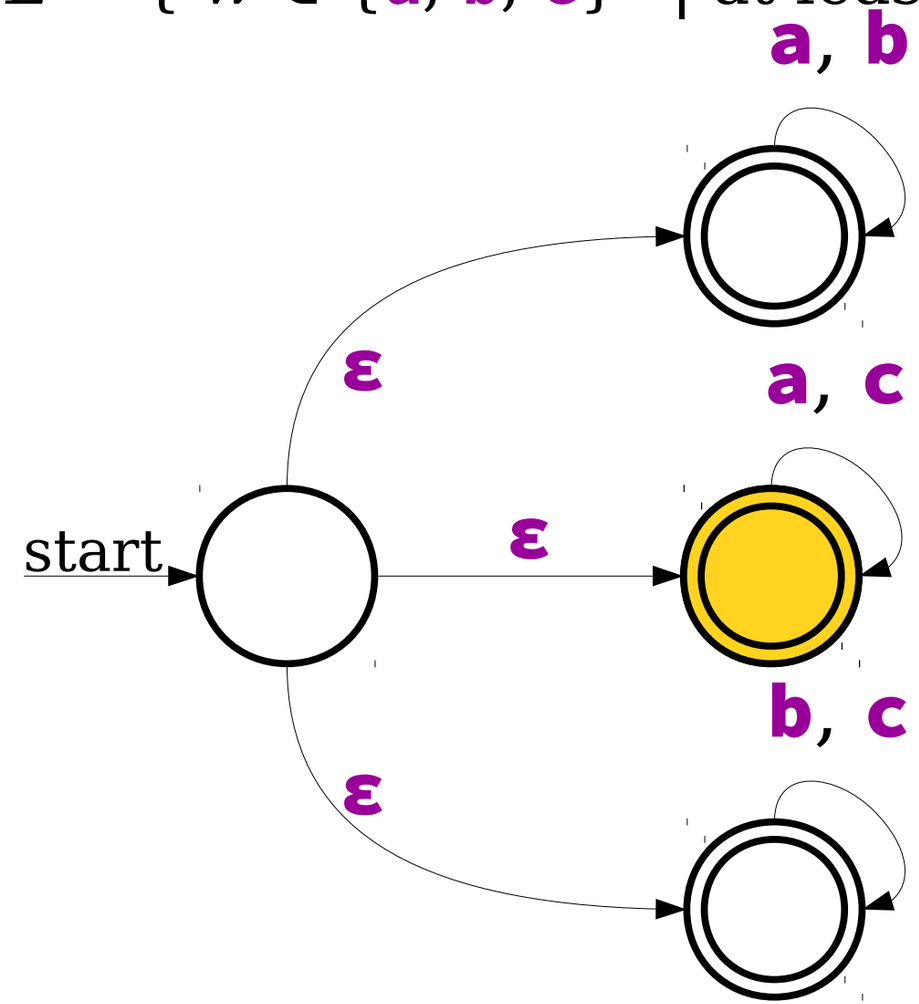

a	c	c	a	c	c

Just how powerful are NFAs?

Next Time

- ***The Powerset Construction***
  - So beautiful. So elegant. So cool!
- ***More Closure Properties***
  - Other set-theoretic operations.
- ***Language Transformations***
  - What's the deal with the notation Σ*?